

A diagnosis of thickness fluxes in an eddy-resolving model

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All figures are available in electronic format.

ABSTRACT

We use output from a $1/12^\circ$ eddy-resolving model of the North Atlantic Ocean to estimate values for the thickness diffusivity, κ , appropriate to the Gent and McWilliams parameterization. Three different procedures are used to remove a rotational flux prior to the estimation of the thickness diffusivity. Using the raw fluxes (no rotational flux removed), large negative values (exceeding $-5000 \text{ m}^2/\text{s}$) of κ are diagnosed, particularly in the Gulf Stream region and in the equatorial Atlantic and large positive values (exceeding $5000 \text{ m}^2/\text{s}$) in the subtropical gyre. Removing a rotational flux based on the suggestion of Marshall and Shutts (1981) leads to reduction of both the negative and positive values. Removing a rotational flux based on the more general theory of Medvedev and Greatbatch (2004) reduces the regions with negative κ , but large positive values for κ exceeding $5000 \text{ m}^2/\text{s}$ are still found in the subtropical gyre. We also estimate the rotational flux using an optimization technique. In this case, the absolute value of κ is reduced almost everywhere by construction, but at the expense of increasing the region where negative values are diagnosed. The regions where κ is negative consistently correspond with regions where eddies are acting to increase, rather than decrease (as in baroclinic instability) the mean available potential energy. Therefore, negative values of κ are physically justified and should be implemented in parameterizations.

The theoretical development also reveals a previously neglected component (ν) of the bolus velocity associated with the horizontal flux of buoyancy along, rather than across the mean buoyancy contours. All the estimates show a similar large-scale pattern for ν . It can be interpreted as a streamfunction for eddy-induced advection rather than diffusion of mean isopycnal layer thickness, or equivalently, as advection rather than diffusion of mean stretching vorticity. Comparing ν with a mean streamfunction shows that it is about 10% of the mean in mid-latitudes and even larger than the mean in the tropics.

1 Introduction

The parameterization for the advective effects of meso-scale activity on tracers in the ocean, as first proposed in the literature by Gent and McWilliams (1990) (GM hereafter), is currently applied in almost any state-of-the-art coarse resolution general circulation ocean model. In this parameterization a number with units of a diffusivity shows up, which has to be specified. In most models, a value of about $1000 \text{ m}^2/\text{s}$ is used, sometimes higher in the upper ocean and lower below the thermocline. Since it is possible to interpret the GM parameterization as an (almost Fickian) diffusion of isopycnal layer thickness, this parameter is also called the “thickness diffusivity”. Because of the sparseness of observations of interior meso-scale activity it appears to be difficult to estimate this parameter

directly from available observations. On the other hand, there have been some attempts to infer the thickness diffusivity from the synthetic data of eddy resolving general circulation models, e. g. [Rix and Willebrand \(1996\)](#); [Jochum \(1997\)](#); [Bryan et al. \(1999\)](#); [Nakamura and Chao \(2000\)](#); [Roberts and Marshall \(2000\)](#); [Drijfhout and Hazeleger \(2001\)](#); [Peterson and Greatbatch \(2001\)](#); [Soloviev et al. \(2002\)](#).

However, these attempts have only been partly successful even in determining the overall gross magnitude of the thickness diffusivity in the ocean. [Rix and Willebrand \(1996\)](#), using 5 year averaged moments of meso-scale activity from an eddy-permitting model of the North Atlantic suggest that longer time series are needed to get stable estimates. [Jochum \(1997\)](#) using 20 year averages of eddy fluxes from the same model came to the same conclusion. Both were only able to give a rough estimate of a mean thickness diffusivity in the southern part of the subtropical North Atlantic thermocline, excluding western boundary current regions.

[Bryan et al. \(1999\)](#) pointed out that it might be necessary to subtract (or add) rotational eddy fluxes from the “raw” eddy fluxes to get meaningful results in terms of thickness diffusivity. Even with perfect statistics of the meso-scale activity in the real ocean, one might still obtain incorrect results without considering rotational fluxes, making the task of determining the value of the “real” thickness diffusivity for use in a general circulation model very difficult.

On the other hand, several authors ([Roberts and Marshall, 2000](#); [Drijfhout and Hazeleger, 2001](#); [Jayne and Marotzke, 2002](#)) perform a Helmholtz-decomposition of the eddy fluxes into divergent and rotational parts. Using this decomposition, however, it is assumed that the part of the fluxes that can be used for eddy parameterizations is completely irrotational, an overly restrictive assumption not imposed, for example, in [Eden et al. \(2005\)](#). Moreover, the decomposition depends on the definition of boundary conditions for divergent and rotational parts, which cannot be defined in a unique manner. This choice for rotational fluxes remains therefore ambiguous. [Nakamura and Chao \(2000\)](#) try to infer patterns of thickness diffusivity in an eddy-permitting model of the North Atlantic by considering the divergence of the eddy fluxes only, which eliminates the problem of rotational fluxes by construction, but yield rather noisy results and no clear sign or order of magnitude of the thickness diffusivity.

Using data from a realistic eddy-resolving model of the North Atlantic, we test two choices for the rotational fluxes, proposed by [Marshall and Shutts \(1981\)](#) and a more general approach by [Medvedev and Greatbatch \(2004\)](#). As an alternative to specifying a rotational flux based on theoretical considerations, we also estimate rotational fluxes using minimizations with respect to the value of thickness diffusivity, but including information about rotational fluxes. In all our estimates we show the thickness diffusivity (κ), related to the horizontal eddy flux of buoyancy down the horizontal gradient of mean density, and a new parameter (ν) related to a previously neglected component of the eddy flux

along, rather than across, the mean density contours. We argue that this part of the flux, which is not included in GM, might also be important for large-scale flows.

Following this introduction, we present and discuss the GM-parameterization in section 2 and show eddy fluxes from an eddy-resolving realistic model of the North Atlantic in section 3. In section 4, we review the rotational eddy flux choice of [Marshall and Shutts \(1981\)](#) and show thickness diffusivity κ and ν using this rotational flux and using the raw eddy fluxes. In section 5, we introduce the rotational flux of [Medvedev and Greatbatch \(2004\)](#) and show its consequences on κ and ν followed by section 6, in which we introduce a minimal condition to constrain κ , ν and the rotational flux and show results from different minimizations. The last section summarizes and discusses our conclusions.

2 A generalization of the GM-parameterization

In this section we briefly derive the GM-parameterization in the framework of the Transformed Eulerian Mean ([Andrews and McIntyre, 1976](#)). For simplicity, we neglect any complication of a non-linear equation of state and use b as buoyancy with $b = -g(\rho - \rho_0)/\rho_0$. The mean buoyancy budget is given by

$$\bar{b}_t + \bar{\mathbf{u}} \cdot \nabla \bar{b} + \nabla \cdot \mathbf{F} = \bar{Q} \quad (1)$$

with buoyancy $b = \bar{b} + b'$, velocity $\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$ and \bar{Q} the mean diabatic forcing. $\mathbf{F} = \overline{\mathbf{u}'b'}$ denotes the eddy buoyancy flux, primes deviations from a temporal mean denoted by an overbar. Now we decompose \mathbf{F} as

$$\mathbf{F} = -K\nabla\bar{b} + \mathbf{B} \times \nabla\bar{b} + \nabla \times \boldsymbol{\theta} \quad (2)$$

with a *diapycnal* diffusivity $K = -|\nabla\bar{b}|^2(\mathbf{F} - \nabla \times \boldsymbol{\theta}) \cdot \nabla\bar{b}$ and a rotational flux given by the vector potential $\boldsymbol{\theta}$, which drops out in the divergence of \mathbf{F} and which therefore does not show up in Eq. (1). Following GM, we assume for now and for the remainder of this paper, that meso-scale activity is completely adiabatic so that $K = 0$. Note that this view of the effect of eddy activity in the ocean has been recently under debate, see e.g. ([Radko and Marshall, 2004](#); [Eden et al., 2005](#)). We also begin by ignoring the rotational vector potential, so that $\boldsymbol{\theta} = 0$, in this section. Note, however, that we will define certain non-zero vector potentials $\boldsymbol{\theta}$ in the following sections for reasons explained below. Since

$$\nabla \cdot \mathbf{F} = \nabla \times \mathbf{B} \cdot \nabla\bar{b} \equiv \mathbf{u}^* \cdot \nabla\bar{b} \quad (3)$$

the curl of \mathbf{B} acts like an eddy-induced, three-dimensional advection velocity \mathbf{u}^* which adds to the Eulerian mean velocity $\bar{\mathbf{u}}$ in Eq. (1) and is often called the “bolus velocity”, while the sum of bolus velocity and Eulerian mean velocity is sometimes called the “residual velocity”.

The second part of the flux decomposition in Eq. (2) introduces a vector streamfunction \mathbf{B} for the bolus velocity \mathbf{u}^* , which can be written as (using the gauge condition $\mathbf{B} \cdot \nabla \bar{b} = 0$)

$$\mathbf{B} = -|\nabla \bar{b}|^{-2} \mathbf{F} \times \nabla \bar{b} = |\nabla \bar{b}|^{-2} \begin{pmatrix} \overline{v'b'} \bar{b}_z - \overline{w'b'} \bar{b}_y \\ \overline{w'b'} \bar{b}_x - \overline{u'b'} \bar{b}_z \\ \overline{u'b'} \bar{b}_y - \overline{v'b'} \bar{b}_x \end{pmatrix} \approx \bar{b}_z^{-1} \begin{pmatrix} \overline{v'b'} \\ -\overline{u'b'} \\ \overline{u'b'} s_2 - \overline{v'b'} s_1 \end{pmatrix} \quad (4)$$

where $\mathbf{s} = (s_1, s_2)^T = \bar{b}_z^{-1} \nabla_h \bar{b}$ denotes the isopycnal slope vector. Note that the last approximation in Eq. (4) is valid only for $|\bar{b}_z| \gg |\nabla_h \bar{b}|$ (we care about this situation only and ignore complications in e. g. a surface mixed layer).

The streamfunction \mathbf{B} is expressed in terms of the eddy buoyancy fluxes which have to be parameterized for the use in a coarse resolution model. In the GM parameterization this is achieved by assuming that the horizontal eddy fluxes $\overline{\mathbf{u}'_h b'}$ are directed down the horizontal gradient of the mean buoyancy, i. e. $\overline{\mathbf{u}'_h b'} \approx -\kappa \bar{\nabla}_h \bar{b}$, with $\kappa > 0$. Here, we express the eddy fluxes as

$$\mathbf{F}_h \equiv \overline{\mathbf{u}'_h b'} = -\kappa \nabla_h \bar{b} - \nu \nabla_h \bar{b} \quad (5)$$

with $\kappa = -|\nabla_h \bar{b}|^{-2} \mathbf{F}_h \cdot \nabla_h \bar{b}$ and $\nu = -|\nabla_h \bar{b}|^{-2} \mathbf{F}_h \cdot \nabla_h \bar{b}$. The operator ∇_h is given by $\nabla_h = (-\frac{\partial}{\partial y}, \frac{\partial}{\partial x})^T$, i. e. a shorthand for $\mathbf{k} \times \nabla_h$ (the vector subscript ∇_h shall denote anti-clockwise rotation of a horizontal vector by 90°). Note that the flux decomposition in Eq. (5) is always possible as long as $|\nabla_h \bar{b}| \neq 0$, while the Fickian diffusion Ansatz $\overline{\mathbf{u}'_h b'} \approx -\kappa \bar{\nabla}_h \bar{b}$ as used in GM is an assumption. We shall show below, that over wide regions of the North Atlantic the along-gradient part of the fluxes, related to ν , is as large as the cross-gradient part of the flux, related to κ . It should be noted that \mathbf{B} and consequently κ and ν depend on the choice of the rotational vector potential $\boldsymbol{\theta}$ (which we have set to zero for now) an issue explored below.

Using the expression Eq. (5) we obtain for the streamfunction of the bolus velocity in Eq. (4)

$$\mathbf{B} = \kappa \begin{pmatrix} -s_2 \\ s_1 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} -s_1 \\ -s_2 \\ s_2^2 + s_1^2 \end{pmatrix} \quad (6)$$

while the bolus velocity \mathbf{u}^* itself is given by

$$\nabla \times \mathbf{B} = \begin{pmatrix} -(\kappa s_1)_z \\ -(\kappa s_2)_z \\ (\kappa s_1)_x + (\kappa s_2)_y \end{pmatrix} + \begin{pmatrix} (\nu s_2)_z \\ -(\nu s_1)_z \\ (-\nu s_2)_x + (\nu s_1)_y \end{pmatrix} + \begin{pmatrix} \frac{\partial}{\partial y} (\nu s_2^2 + \nu s_1^2) \\ -\frac{\partial}{\partial x} (\nu s_2^2 + \nu s_1^2) \\ 0 \end{pmatrix} \quad (7)$$

It is the first term on the r. h. s of equation Eq. (7) which is added in GM to the mean velocity $\bar{\mathbf{u}}$ in coarse resolution models to advect tracers. The parameter κ denotes thickness diffusivity and is usually chosen of the order of $1000 \text{ m}^2/\text{s}$. The assumption that κ is positive ensures that the parameterization releases available potential energy from the mean state (Gent et al., 1995).

However, what is the meaning of the parameter ν and its related part of the bolus velocity? That part of the bolus velocity associated with ν does not figure in the GM parameterization, and is a new aspect of the bolus velocity, even though it emerges quite naturally from our analysis. Some insight into the nature of the parameter ν is gained considering the averaged density perturbation (ρ) budget in the quasi-geostrophic approximation

$$\bar{\rho}_t + \nabla_h \cdot (\bar{\mathbf{u}}_g \bar{\rho}) + R_z \bar{w} = -\nabla_h \cdot \mathbf{F}_g \quad (8)$$

where the geostrophic, zero-order velocities are given by the geostrophic streamfunction $\mathbf{u}_g = \nabla_{\perp} \Psi$, the background stratification by $R(z)$ and the (horizontal) eddy fluxes by $\mathbf{F}_g = \overline{\mathbf{u}'_g \rho'}$. In accordance to the quasi-geostrophic approximation, we have decomposed the full density as $R(z) + \rho$ and the perturbation density ρ again into time average and perturbation as $\rho = \bar{\rho} + \rho'$. The diabatic forcing Q has been neglected in Eq. (8) and for \mathbf{F}_g the above flux decomposition given by Eq. (5) is used again

$$\bar{\rho}_t + (\bar{\mathbf{u}}_g - \nabla_{\perp} \nu) \cdot \nabla_h \bar{\rho} + R_z \bar{w} = \nabla_h \cdot \kappa \nabla \bar{\rho} \quad (9)$$

The density budget Eq. (9) can be transformed into a budget of mean isopycnal layer thickness given by $\bar{h} = \frac{\partial}{\partial z} \left(\frac{\bar{\rho} + R}{R_z} \right)$

$$\bar{h}_t + \nabla_{\perp} (\bar{\Psi} + \nu) \cdot \nabla_h \bar{h} + \bar{w}_z = \nabla_h \cdot \kappa \nabla_h \bar{h} \quad (10)$$

where we have assumed that κ and ν are independent of depth. Note that [Gent et al. \(1995\)](#) also need to assume that $\frac{\partial}{\partial z} \kappa = 0$ to relate κ to thickness diffusion in their parameterization. We see that there is eddy-induced diffusion of mean thickness given by the κ term; therefore κ is often called thickness diffusivity. In addition, however, there is eddy-induced advection of mean thickness related to the ν term. Note that ν acts like a streamfunction which adds to the mean streamfunction $\bar{\Psi}$ accounting for advective eddy effects on the mean thickness. This is the meaning of the ν -related bolus velocity. This interpretation is similar to [Roberts and Marshall \(2000\)](#), who consider the effect of the along-gradient part of the eddy tracer fluxes as being akin to advection of mean tracer extrema in the horizontal plane (compare their Fig.8).

Next we show that in the eddy variance balance of ρ , which gives the budget of (available) eddy potential energy in the quasi-geostrophic approximation, the downgradient flux related to κ shows up as a dissipation of eddy potential energy (as long as κ is positive) while ν does not figure in that equation. Eddy potential energy in the quasi-geostrophic approximation is $-\bar{\phi}(gR_z)^{-1}$ where the eddy variance $\bar{\phi} = \bar{\rho'^2}/2$ is given by

$$\bar{\phi}_t + \nabla_h \cdot \overline{\mathbf{u}_h \phi} = \kappa |\nabla_h \bar{\rho}|^2 - R_z \overline{\rho' w'} \quad (11)$$

We have set again $Q = 0$ and have used again the flux decomposition Eq. (5). Integrating over a closed domain, the advective terms in Eq. (11) drop out and an integral constraint on κ is left, while ν does not figure in that equation. Therefore, there is no effect on eddy potential energy associated with the ν -related part of the eddy fluxes which means, in turn, that there is no integral constraint on the sign of ν as has been argued for thickness diffusivity κ .

Following Eden et al. (2005) introducing a rotational flux in the flux decomposition i. e. $\mathbf{F}_g = -\kappa \nabla_h \bar{\rho} - \nu \nabla_{\perp} \bar{\rho} + \nabla_{\perp} \theta_{gg}$ leads to a local definition of κ and ν as we show now. Note that the rotational flux $\nabla_{\perp} \theta_{gg}$ drops out taking the divergence in Eq. (8), but does show up in the variance equation for $\bar{\phi}$. Eden et al. (2005) extended the discussion using a full hierarchy of budgets of moments of ρ' (their TRM-G case), which can be used to write the following local expression for κ

$$\kappa |\nabla_h \bar{\rho}|^2 R_z^{-1} = \overline{\rho' w'} - \frac{1}{2} \mathcal{D}(\overline{\rho'^2 w}) + \frac{1}{3!} \mathcal{D}^2(\overline{\rho'^3 w}) - \frac{1}{4!} \mathcal{D}^3(\overline{\rho'^4 w}) + \dots \quad (12)$$

with the operator $\mathcal{D}() = \nabla_h \cdot \nabla_h \bar{\rho} |\nabla_h \bar{\rho}|^{-2}()$ and a similar expansion for the streamfunction ν . Thus, correlations between vertical velocity and density are related to the thickness diffusivity κ . Note that the leading term in the expression, $\overline{\rho' w'}$, appears in the eddy potential energy budget Eq. (11) and is usually related to baroclinic instability. We show below that in the numerical model this term is usually negative, indicating the release of eddy potential energy by the eddies and implying positive κ , but that regions exist where $\overline{\rho' w'}$ is positive, indicating that eddies are acting to increase the mean potential energy, and implying negative κ .

Note that we use the quasi-geostrophic approximation here only to connect to available potential energy and potential vorticity (see below). Transforming the averaged density budget Eq. (1) using the flux decomposition Eq. (5) and the continuity equation to a system using \bar{b} as vertical coordinate also shows the advective nature of the parameter ν and the diffusive nature of the parameter κ . However, to make the analogy complete one has also to assume that κ and ν are independent of depth. Thus, for the general case, the κ and ν related fluxes are acting “like” thickness diffusion and advection but are not identical.

A similar effect of ν as in the thickness balance shows up in the budget of quasi-geostrophic potential vorticity q . Combining the averaged density budget with the linear, averaged first order momentum and continuity equations given by

$$-\beta y \bar{\Psi}_x - f_0 \bar{v} = -\bar{p}_x \quad , \quad -\beta y \bar{\Psi}_y + f_0 \bar{u} = -\bar{p}_y \quad , \quad \nabla_h \cdot \bar{\mathbf{u}}_h + \bar{w}_z = 0 \quad (13)$$

yields, omitting some algebra here,

$$\frac{\partial}{\partial t} \bar{q} + \nabla_{\perp} \bar{\Psi} \cdot \nabla_h \bar{q} + \nabla_{\perp} \nu \cdot \nabla_h \bar{q}_s = \nabla_h \cdot \kappa \nabla_h \bar{q}_s \quad (14)$$

where we have again assumed that κ and ν are independent of depth. Following usual convention, we have defined the potential vorticity as $q = \beta y + (\frac{f_0^2}{N^2} \Psi_z)_z = \beta y + q_s$. We see that while thickness diffusion related to κ acts like diffusion of mean stretching vorticity, q_s , in the potential vorticity equation (Greatbatch and Lamb, 1990), the eddy-induced advection of thickness related to ν acts like advection of mean stretching vorticity, where ν resembles again a streamfunction. We stress that there is no reason to neglect the latter in a consistent eddy parameterization. An advective eddy effect can be as important as an diffusive effect, an argument originally proposed by Gent et al. (1995). For these reasons we diagnose and discuss the ν part as well as the downgradient κ part of the eddy fluxes in our eddy-resolving model results.

3 Eddy fluxes in the eddy resolving model

To illustrate how well the Fickian assumption $\overline{\mathbf{u}'_h b'} \approx -\kappa \bar{\nabla}_h b$ assumed by GM actually holds up, we show $\overline{\mathbf{u}'_h b'}$ and \bar{b} in the main thermocline in an realistic eddy resolving model (1/12°) of the North Atlantic. The horizontal resolution of this model is about 10 km at the equator decreasing to about 5 km in high latitudes. The model domain extents from 20°S to 70°N with open boundaries (Stevens, 1990) at the northern and southern boundaries and with a restoring zone in the eastern Mediterranean Sea. There are 45 vertical geopotential levels with increasing thickness with depth, ranging from 10 m at the surface to 250 m near the maximal depth of 5500 m. The model is based on a rewritten version¹ of MOM2 (Pacanowski, 1995), is similar to the one used in Eden and Böning (2002) and Brandt et al. (2005) and identical to the one used in Dengler et al. (2004). All moments shown here are averaged over 5 years following a spinup phase of 10 year model integration using climatological surface forcing which is identical to the forcing used in Eden and Böning (2002) and Dengler et al. (2004).

Fig. 1 a) displays the situation in the subtropical gyre. Here, the eddy fluxes appear indeed to be down the gradient of \bar{b} . In contrast, Fig. 1 b) shows that in the area of the North Atlantic Current the dominant direction of the eddy fluxes appears to be more perpendicular to $\nabla_h \bar{b}$. Moreover, the (smaller) downgradient part of the flux shows no clear sign as in the subtropical gyre.

However, even if the fluxes do not appear to be downgradient, this does not mean that the GM-parameterization is incorrect. As noted at the beginning of section 2, it is possible to add (or remove) a rotational flux $\boldsymbol{\theta}$ to the eddy buoyancy fluxes. This rotational flux is not affecting the mean density budget, but is affecting the definition of the parameter κ and ν in Eq. (5). It might be possible to remove a rotational flux from \mathbf{F} such that the downgradient GM parameterization remains valid. In

¹The numerical code together with all configurations used in this study can be accessed at <http://www.ifm.uni-kiel.de/fb/fb1/tm/data/pers/ceden/spflame/index.html>.

the following we will test two choices for such a rotational flux coming from theoretical considerations and also a optimization technique to determine the rotational flux.

4 Rotational eddy fluxes: Marshall and Shutts (1981)

A simple choice for rotational eddy fluxes was proposed by [Marshall and Shutts \(1981\)](#). They consider the eddy buoyancy variance equation in which they assume a steady state, adiabatic flow regime and neglect advection by the perturbation flow to obtain the simple balance

$$\bar{\mathbf{u}} \cdot \nabla \bar{\phi} + \mathbf{F} \cdot \nabla \bar{b} \approx 0 \quad (15)$$

where $\bar{\phi} = \overline{b'^2}/2$ denotes eddy buoyancy variance. They further assume that the mean flow can be described by a quasi-geostrophic streamfunction (and hence is in the horizontal plane) and flows parallel to the mean buoyancy (\bar{b}) contours so that $\bar{\mathbf{u}}_h = \nabla_{\perp} B_m$, with $B_m = B_m(\bar{b}, z)^2$. Using this expression in the balance of the eddy buoyancy variance they obtain

$$\overline{w'b'} \bar{b}_z + (\mathbf{F}_h - \frac{\partial}{\partial b}(B_m) \nabla_{\perp} \bar{\phi}) \cdot \nabla_h \bar{b} = 0 \quad (16)$$

which motivates [Marshall and Shutts \(1981\)](#) to set the rotational part of the horizontal eddy flux to $\mathbf{F}_R = (\nabla \times \boldsymbol{\theta})_h = \frac{\partial}{\partial b}(B_m) \nabla_{\perp} \bar{\phi}$. The rotational part of the eddy fluxes (\mathbf{F}_R) cancels the effect of the mean advection of eddy variance in Eq. (16). Note that after removing the rotational flux, the component of the remaining part of \mathbf{F}_h in the direction of $\nabla_h \bar{b}$ balances the production term, propotional to $\overline{w'\rho'}$, and determines the sign of κ , a relation we diagnose from our model results below and which we already noted in section 2.

We define in a first attempt the rotational part of the eddy fluxes \mathbf{F} , following [Marshall and Shutts \(1981\)](#), as the projection of \mathbf{F} perpendicular to the horizontal gradient of the eddy variance, i. e. as

$$\mathbf{F}_R = \frac{\partial}{\partial b}(B_m) \nabla_{\perp} \bar{\phi} \quad \text{with} \quad \frac{\partial}{\partial b}(B_m) = -|\nabla_h \bar{\phi}|^{-2} \overline{\mathbf{u}'_h \bar{b}'} \cdot \nabla_{\perp} \bar{\phi} \quad (17)$$

and subtract \mathbf{F}_R from the raw fluxes \mathbf{F} to parameterize the streamfunction \mathbf{B} . For this choice, the thickness diffusivity κ and ν are given by

$$\kappa_{ms} = -|\nabla_h \bar{b}|^{-2} (\overline{\mathbf{u}'_h \bar{b}'} - \frac{\partial}{\partial b}(B_m) \nabla_{\perp} \bar{\phi}) \cdot \nabla_h \bar{b} \quad (18)$$

$$\nu_{ms} = -|\nabla_h \bar{b}|^{-2} (\overline{\mathbf{u}'_h \bar{b}'} - \frac{\partial}{\partial b}(B_m) \nabla_{\perp} \bar{\phi}) \cdot \nabla_{\perp} \bar{b} \quad (19)$$

²The dependency of B_m on \bar{b} is clear since the flow follows \bar{b} -contours. However, this dependency on b might change with depth, therefore $B_m = B_m(\bar{b}, z)$.

where we have labelled the parameters κ and ν to indicate that they are derived using Eq. (18) and Eq. (19).

The choice of Marshall and Shutts (1981) is supported looking again at the eddy fluxes in the North Atlantic Current region. Fig. 2 displays the eddy fluxes \mathbf{F} and the eddy variance $\bar{\phi}$ in that region. It is indeed evident in this region with high levels of eddy variance, that a large part of the eddy flux is circulating along contours of $\bar{\phi}$, consistent with the choice of Marshall and Shutts (1981).

Now we look at the actual thickness diffusivities κ and their counterpart ν (which is not included in the GM parameterization) as diagnosed from the eddy resolving model. We start with κ and ν calculated from the “raw” eddy fluxes, i. e. taking $\boldsymbol{\theta} = \mathbf{0}$, as

$$\kappa_{raw} = -|\nabla_h \bar{b}|^{-2} \overline{\mathbf{u}'_h b'} \cdot \nabla_h \bar{b} \quad \text{and} \quad \nu_{raw} = -|\nabla_h \bar{b}|^{-2} \overline{\mathbf{u}'_h b'} \cdot \nabla_{\perp} \bar{b} \quad (20)$$

where we have again labelled the parameters κ and ν to indicate that they are derived from Eq. (20). Fig. 3 shows κ_{raw} and ν_{raw} over the whole North Atlantic in the main thermocline. In the subtropics, distant from any boundary currents, κ_{raw} is indeed positive although very large with values exceeding $5000 \text{ m}^2/\text{s}$, somewhat larger than the canonical value of about $1000 \text{ m}^2/\text{s}$ used in coarse resolution models. Note that this is the region in which Rix and Willebrand (1996) estimated a thickness diffusivity of about $1000 \text{ m}^2/\text{s}$ in their eddy-permitting model. It appears that higher horizontal resolution and more vigorous eddy activity in our eddy-resolving model yields higher thickness diffusivities in this region, using the raw eddy fluxes for the estimation of κ .

Approaching the equator, κ_{raw} remains large near the western boundary but changes its sign several times. In the Gulf Stream region, there is also a large area with negative values less than $-5000 \text{ m}^2/\text{s}$, followed by changes in sign further downstream in the North Atlantic Current. The appearance of large negative values may come as a surprise, since we expect the Gulf Stream and North Atlantic Current to be baroclinically unstable regions, where the available potential energy of the mean state is being released, and κ in general should therefore be positive. To give insight into this issue, Fig. 4 shows the vertical eddy buoyancy flux in the main thermocline as an indicator for local energy transfer from mean potential energy to eddy kinetic energy by baroclinic instability (Lorenz, 1955). Over large areas, $\overline{w'b'}$ is negative, denoting indeed release of mean potential energy to eddy kinetic energy, as we expect. However, in the region immediately to the south of the Gulf Stream and further downstream, where eddies are decaying rather than being generated, and a few degrees latitude north and south of the equator (consistent with recent analysis of eddy feedbacks on equatorial currents by Jochum and Malanotte-Rizzoli (2004)), $\overline{w'b'}$ is positive and very large, pointing to the opposite mechanism. The regions with positive $\overline{w'b'}$, i. e. regions with transfer from eddy kinetic energy to the mean stratification, are coinciding roughly with regions of negative κ_{raw} . Thus negative

κ_{raw} 's appear to be related to regions in which energy contained in meso-scale activity is feeding back to the mean flow.

The picture is complemented by a look at ν_{raw} shown in Fig. 3 b). Note that ν_{raw} should vanish or at least be small for the downgradient GM parameterization to be valid (using the raw eddy fluxes). It is not, in fact it is of the same order of magnitude as κ_{raw} and shows large-scale gradients. A notable feature of ν_{raw} is the tendency for it to be positive in the northern half of the subtropical gyre and negative in the southern half of the subtropical gyre.

Eq. (10) shows that ν acts in the quasi-geostrophic approximation as a streamfunction which adds to the mean geostrophic flow and which advects mean thickness (in contrast to diffusion of mean thickness related to κ). Alternatively, Eq. (14) relates ν to eddy-induced advection of mean stretching vorticity. It's meaning is again a streamfunction for the eddy-induced advection. Fig. 5 shows an estimate for the geostrophic streamfunction Ψ_m of the mean flow, simply by solving $\nabla_h^2 \Psi_m = \nabla_h \cdot \bar{\mathbf{u}}_h$ using $\bar{\mathbf{u}}$ from the model output. The values of Ψ_m ranges about $\pm 50000 \text{ m}^2/\text{s}$, thus the ν -related flow is of the order of 10% of the mean flow in our eddy-resolving model which means that there is a significant eddy-induced part of the advection velocity. Note that assuming an uniform value of Ψ_m over a layer with 1000 m thickness yields maximal transports of $\pm 50 \times 10^6 \text{ m}^3/\text{s} = 50 \text{ Sv}$ to which ν would contribute by more than $\pm 5 \text{ Sv}$.

The large-scale pattern of ν , i. e. positive in the northern half of the subtropical gyre and negative in the southern half, implies westward advection in the center of the gyre, reminiscent of westward propagation by long, non-dispersive, baroclinic Rossby waves. Indeed, the implied advective velocity is of similar magnitude, i. e. $\mathcal{O}(1 \text{ cm/s})$, as the propagation speed for long, baroclinic Rossby waves. However, north of the subtropical gyre, the eddy-induced advection tends to be eastward with the consequence that in the Gulf Stream and North Atlantic Current region, the eddy-induced advection is supporting the mean flow, reminiscent of eddies advected by the mean flow. In the tropical Atlantic the pattern of ν is more complicated, but here ν and its gradients, i. e. the eddy-induced advection of thickness, is of the same order of magnitude and sometimes even larger as Ψ_m and its gradients, i. e. the mean advection.

Subtracting rotational fluxes following Marshall and Shutts (1981) before projecting the (divergent part of the) eddy fluxes onto $\nabla_h \bar{b}$ gives different results in terms of κ and ν . Fig. 6 shows thickness diffusivities κ_{ms} and their counterpart ν_{ms} estimated from Eq. (18) and Eq. (19). First, we note that the patterns of κ and ν are now less noisy compared to Fig. 3. In large parts of the subtropical gyre, κ_{ms} is now positive and about $1000\text{-}3000 \text{ m}^2/\text{s}$, more comparable but still larger than the canonical values used in coarse resolution models. On the other hand, we see still a large area of negative values of κ_{ms} in the Gulf Stream region (although with smaller magnitudes). This comes along with smaller

values of ν_{ms} than before with alternating sign in the subtropical gyre. Note that the tendency for ν_{ms} to be positive north of about $30^\circ N$ and negative south of about $30^\circ S$ is even clearer than before in Fig. 3. However, approaching the equator, κ_{ms} still tends to change sign and the pattern of ν_{ms} hardly changes compared to ν_{raw} . We might argue that this is due to the geostrophic assumption in the approach by Marshall and Shutts (1981), a drawback which is relaxed using the definition for rotational eddy fluxes of Medvedev and Greatbatch (2004).

5 Rotational eddy fluxes: Medvedev and Greatbatch (2004)

Marshall and Shutts (1981) make several assumptions (e. g. that the mean flow is parallel to the mean buoyancy contours) and neglect several terms in the eddy variance equation (e. g. the triple correlation term), which might play a significant role, such that the results in terms of the estimated thickness diffusivity might be obscured. A generalization of the idea is given by Medvedev and Greatbatch (2004). Consider again the variance equation

$$\nabla \cdot \overline{\mathbf{u}\phi} + \nabla \times \boldsymbol{\theta} \cdot \nabla \bar{b} = \overline{Q'b'} \quad (21)$$

where, as before, we have neglected diabatic effects associated with the eddies (i. e. $K = 0$) and changes in eddy variance (i. e. $\frac{\partial}{\partial t}\phi = 0$) and where $\overline{Q'b'}$ corresponds to dissipation of variance. Now assume that the rotational gauge potential has only one non-zero component chosen as $\boldsymbol{\theta} = (0, 0, \theta_3)^T$ and decompose the horizontal advection of eddy variance into components along and across the horizontal gradient of the mean buoyancy, i. e. $\overline{\mathbf{u}_h\phi} = \alpha \nabla_h \bar{b} + \gamma \nabla_\perp \bar{b}$ which yields

$$(\overline{w\phi})_z + \nabla_h \cdot \alpha \nabla_h \bar{b} + \nabla_h \gamma \cdot \nabla_\perp \bar{b} - \nabla_\perp \theta_3 \cdot \nabla_h \bar{b} = \overline{Q'b'} \quad (22)$$

Following Medvedev and Greatbatch (2004), it is straightforward to set $\theta_3 = -\gamma$ and the eddy variance equation becomes

$$(\overline{w\phi})_z + \nabla_h \cdot \alpha \nabla_h \bar{b} = \overline{Q'b'} \quad (23)$$

thus only the cross isopycnal fluxes of variance in the horizontal plane are left to balance the vertical advection of variance $(\overline{w\phi})_z$ and dissipation $\overline{Q'b'}$ of variance in Eq. (23). The difference to Marshall and Shutts (1981) is that no assumption about the nature of the mean flow was made, e. g. the definition should be valid also for diabatic flows and at the equator. The thickness diffusivity κ_{mg} and its counterpart ν_{mg} for the choice of the rotational flux potential of Medvedev and Greatbatch (2004) are given by

$$\kappa_{mg} = -|\nabla_h \bar{b}|^{-2} (\overline{\mathbf{u}'_h b'} - (\nabla \times \boldsymbol{\theta})_h) \cdot \nabla_h \bar{b} \quad (24)$$

$$\nu_{mg} = -|\nabla_h \bar{b}|^{-2} (\overline{\mathbf{u}'_h b'} - (\nabla \times \boldsymbol{\theta})_h) \cdot \nabla_\perp \bar{b} \quad (25)$$

with

$$\boldsymbol{\theta} = (0, 0, -\gamma)^T \quad \text{and} \quad \gamma = |\nabla_h \bar{b}|^{-2} \overline{\mathbf{u}_h \phi} \cdot \nabla \bar{b} \quad \text{and} \quad (\nabla \times \boldsymbol{\theta})_h = \nabla \gamma \quad (26)$$

Fig. 7 shows thickness diffusivities κ_{mg} and the streamfunction ν_{mg} estimated from Eq. (24) and Eq. (25). Now there are fewer grid points with negative values of κ in the Gulf Stream region and the equatorial Atlantic compared to Fig. 6 and Fig. 3, although the regions with negative κ_{mg} continue to coincide with regions in which $\overline{w'b'}$ is negative, as we expect. On the other hand, the positive values of κ in the subtropical North Atlantic and the eastern flank of the North Atlantic Current have increased, exceeding now $5000 \text{ m}^2/\text{s}$ over wide regions. Interestingly, the ν_{mg} field continues to show the asymmetry between positive (negative) values over the northern (southern) part of the subtropical gyre. The large-scale pattern of ν is almost unaffected by different choices of the rotational flux potential $\boldsymbol{\theta}$, while values of κ do depend on $\boldsymbol{\theta}$.

6 Optimized rotational eddy fluxes

As an alternative to specifying a rotational flux based on theoretical considerations, we now proceed to estimate a rotational flux by minimizing numerically κ and ν and θ_3 in

$$\int dV \text{std}(\kappa)^{-2} (\kappa - \kappa_0)^2 + \text{std}(\nu)^{-2} (\nu - \nu_0)^2 + \text{std}(\theta_3)^{-2} (\theta_3 - \theta_0)^2 = \min \quad (27)$$

under the constraint that κ , ν and θ_3 satisfy $\mathbf{F}_h = -\kappa \nabla_h \bar{b} - \nu \nabla \bar{b} + \nabla \theta_3$ using standard methods (Wunsch, 1996). Since we try to solve an underdetermined system, there are an infinite number of solutions satisfying the flux decomposition, therefore deviations from a priori parameters κ_0 , ν_0 and θ_0 are minimized under the constraint that the parameters are satisfying the flux decomposition. Furthermore, the system is weighted by specifying standard deviations in which the parameters are allowed to vary, i. e. $\text{std}(\kappa, \nu)$ and $\text{std}(\theta_3)$.

It is important to formulate explicitly the question asked by the minimization Eq. (27) to interpret the answer which is obtained. We ask for a decomposition of the horizontal eddy fluxes in terms of the parameters κ , ν and θ_3 , which gives minimal deviations of the parameters from a priori known values. We do not know much about the a priori parameters. What we do know is that κ should be positive in an integral sense, as given from the balance of available potential energy in the quasi-geostrophic approximation, and that θ_3 could be chosen using Marshall and Shutts (1981) or Medvedev and Greatbatch (2004) as a guide.

We start by showing results of a first minimization (MINI-I) using $\kappa_0 = 2000 \text{ m}^2/\text{s}$, $\nu_0 = 0$ and $\theta_0 = -\gamma$ as a priori parameters and using the weights $\text{std}(\kappa, \nu) = 1000 \text{ m}^2/\text{s}$ and $\text{std}(\theta_3) = 10000 \text{ m}^3/\text{s}^3$.

Note that all parameters of the different minimizations discussed here are summarized in Table 1. That means, that we use as a priori information that κ should be similar to the canonical, constant value used in coarse resolution models and that there are rotational eddy fluxes present in the “raw” fluxes as given by the choice of Medvedev and Greatbatch (2004). We minimize deviations of κ from the canonical value, implying that we search for a (more or less) positive, constant value of κ . For ν we simply search for small values, implying that this parameter should be zero. In effect, this choice for the a priori parameters implements some of the currently accepted knowledge about eddy fluxes.

Fig. 8 shows the results of the minimization MINI-I in terms of the optimal κ and ν . As constrained by the minimal condition, the optimal κ is now positive over large regions of the North Atlantic. Only in a few grid points in the Gulf Stream region and 5° north and south of the equator we find negative values of the thickness diffusivity. Note that these regions still coincide with the regions where $\overline{w'b'}$ is negative. The positive values in the subtropical North Atlantic are much smaller than before in Fig. 7; overall the optimal κ ranges between 1000 and 3000 m^2/s similar to the Marshall and Shutts case. The values of the optimal ν have much decreased as well, in general, but are still of the same order of magnitude as κ . Note that we aimed to minimized values of ν in the solution of Eq. (27). Interestingly, ν remains predominantly positive north of about $30^\circ N$ and negative south of this latitude, as we noted in the other cases. Indeed, this asymmetry in ν is a robust feature of our results. We take this as evidence that the ν -related part of the bolus velocity is an important aspect of eddy fluxes.

Fig. 9 displays the optimal rotational gauge potential θ_3 and also $-\gamma$, which was used to estimate κ and ν in Fig. 7 and which serves as the a priori information for θ_3 in MINI-I. While in the tropical Atlantic, both figures are rather similar, it appears from the figure that the minimization alters the a priori rotational gauge potential north of about $30^\circ N$. However, it is the gradient of θ_3 which matters and which does not change much in that region. Note that there is a free, constant value of θ_3 . Largest gradients of similar direction and amplitude show up in both the a priori and the a posteriori rotational potential in Fig. 9 in the Gulf Stream region and further downstream, pointing towards strong westward rotational eddy fluxes in this region.

We show in Fig. 10 results from a second minimization (MINI-II) using the a priori parameter $\kappa_0 = 0$ while all other parameters stay the same as in MINI-I. Note that ν (and θ_3 , not shown) hardly changes displaying again a similar pattern as in all figures before. By construction, values of κ are decreasing in this minimization. In consequence, there are now larger regions with negative values of κ . However, the magnitude of the negative values is less than before for κ in the flux decompositions by Marshall and Shutts (1981) or Medvedev and Greatbatch (2004). Only near the coast of North America at the Gulf Stream separation point a few values more negative than $-5000 m^2/s$ show up. Note that the regions of negative κ still coincide with regions of positive $\overline{w'b'}$. On the other hand,

positive values of κ are dominating even in this flux decomposition, such that the integral constraint on κ to be positive will be satisfied.

In a third minimization (MINI-III), we also set $\theta_0 = 0$ while all other parameters stay the same as in MINI-II. The results in terms of κ , ν and θ_3 (not shown) are very similar to MINI-II, showing that the choice for the rotational eddy fluxes by [Medvedev and Greatbatch \(2004\)](#) yields similar results as the outcome from our minimization constraint by the a priori information of zero rotational eddy flux, pointing towards the need of rotational eddy fluxes. The minimizations show that with zero rotational flux, κ and ν get larger (as in the raw fluxes case shown in Fig. 10).

From the similarity of the large-scale patterns in κ , ν and θ_3 in the minimization and the above sections, we conclude that the previous flux decompositions are in accordance with the results from the minimization. This means that there are no artificially large parameters of some kind in the previous flux decompositions. In particular, the similar patterns of ν in all results and the fact that ν does not vanish in the minimization, denotes that ν is a fundamental aspect of eddy fluxes and is actually needed in a consistent flux decomposition. The same holds for the rotational eddy fluxes.

7 Conclusions

Previous attempts to verify the GM parameterization for meso-scale eddy transport, and to obtain an estimate for the thickness diffusivity, have been thwarted by the presence of large rotational eddy tracer fluxes that have no effect on the mean buoyancy budget, but complicate the interpretation of the individual eddy tracer fluxes. Here, we have used long term averaged output from a $1/12^\circ$ eddy-resolving model of the North Atlantic Ocean to estimate values for the thickness diffusivity, κ , using different procedures to remove a rotational flux prior to the estimation of κ .

a) Thickness diffusivity

Using the raw fluxes (no rotational flux removed), large negative values of κ are diagnosed, particularly in the Gulf Stream region, further downstream in the North Atlantic Current and in the equatorial Atlantic. These regions coincide with regions of energy transfer from eddy kinetic energy to the mean potential energy. Removing a rotational flux based on the suggestion of [Marshall and Shutts \(1981\)](#) is reducing these negative values slightly, while removing a rotational flux based on the more general theory of [Medvedev and Greatbatch \(2004\)](#) leads to smaller regions of negative thickness diffusivities and smaller magnitudes of the negative values, but they are still present.

In the subtropical North Atlantic (positive) values for κ estimated from the raw fluxes are highest exceeding $5000 \text{ m}^2/\text{s}$ in the main thermocline over wide areas. These values are considerably larger compared to previous estimates by [Rix and Willebrand \(1996\)](#); [Jochum \(1997\)](#) using similar models at lower (eddy-permitting) resolution. It appears that higher model resolution yields higher thickness diffusivity as estimated from the raw eddy fluxes. Introducing the rotational flux potential following [Marshall and Shutts \(1981\)](#) is reducing these values to $1000 - 3000 \text{ m}^2/\text{s}$, still higher than canonical values of $1000 \text{ m}^2/\text{s}$ used in coarse resolution models. On the other hand, for the [Medvedev and Greatbatch \(2004\)](#) case, values for κ are increasing again similar to the values found for the raw fluxes in the subtropical North Atlantic.

As an alternative to specifying a rotational flux based on theoretical considerations, we also use an optimization technique to estimate the rotational flux similar to [Rix and Willebrand \(1996\)](#). In this case, the absolute value of κ is reduced almost everywhere by construction. When we get higher (lower) thickness diffusivities in the subtropical gyre by constraining the solution to higher (smaller) a priori values of κ , the regions with negative values of κ get smaller (larger). However, negative κ tends to be always present in regions where energy transfer from eddy kinetic energy to the mean potential energy occurs.

In conclusion, all our estimates show large horizontal variations in the values of the thickness diffusivity and consistently negative values of κ in regions where eddies are feeding the mean flow. A constant value of $1000 \text{ m}^2/\text{s}$ for κ as widely used in non-eddy-resolving models appears therefore to be only a rather rough first order approximation to this spatial dependence of thickness diffusivity. We argue that a better parameterization for thickness diffusivity should take negative values of κ in certain regions into account, although numerical restrictions on negative diffusivities might complicate the implementation in models.

b) ν related part of the fluxes

We began by developing the theoretical basis for the GM parameterization within the framework of the Transformed Eulerian Mean. The analysis reveals a previously neglected aspect of the bolus velocity that is associated with the horizontal flux of buoyancy along, rather than across, the mean buoyancy contours. This part of the flux can be interpreted as eddy-induced advection rather than diffusion of mean isopycnal thickness (related to κ), or equivalently, as advection rather than diffusion of mean stretching vorticity. Since ν does not figure in the balance of available potential energy, there is no integral constraint on the sign of ν . It acts as a streamfunction for advection of mean thickness. Comparing ν with an estimate of the mean geostrophic streamfunction shows that ν is of the order of 10% of the mean flow in mid-latitudes and on the same order of magnitude in the tropics.

For the GM parameterization to be valid, the ν related part of the flux should be zero or small compared to κ . In contrast, all our estimates show values of ν comparable or larger than κ . In particular the fact that ν does not vanish in the minimization, denotes that ν is a fundamental aspect of eddy fluxes and is actually needed for a consistent flux parameterization. We speculate that this previously neglected aspect of eddy tracer fluxes demands attention in parameterizing impacts of meso-scale activity on tracers in non-eddy-resolving models.

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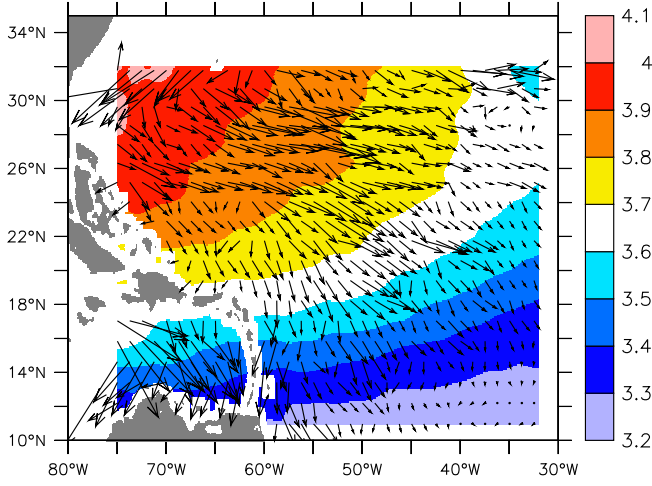
Exp.	κ_0	ν_0	θ_0	$std(\kappa, \nu)$	$std(\theta_3)$
MINI-I	$2000\ m^2/s$	$0\ m^2/s$	$-\gamma$	$1000\ m^2/s$	$10000\ m^3/s^3$
MINI-II	$0\ m^2/s$	$0\ m^2/s$	$-\gamma$	$1000\ m^2/s$	$10000\ m^3/s^3$
MINI-III	$0\ m^2/s$	$0\ m^2/s$	$0\ m^3/s^3$	$1000\ m^2/s$	$10000\ m^3/s^3$

Table 1: Parameter for the different minimizations discussed in section 6.

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a)



b)

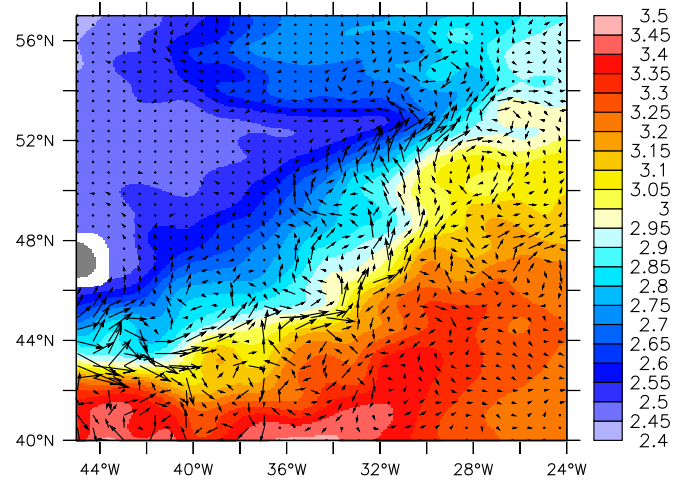


Figure 1: Horizontal eddy fluxes ($\overline{u'_h b'}$, arrows) and mean buoyancy \bar{b} (shading, in m/s^2) in an eddy resolving model of the North Atlantic ($1/12^\circ$ resolution) in 300m depth. Shown are 5 year averages for two different regions. $\overline{u'_h b'}$ and \bar{b} are spatially averaged over 12 grid points.

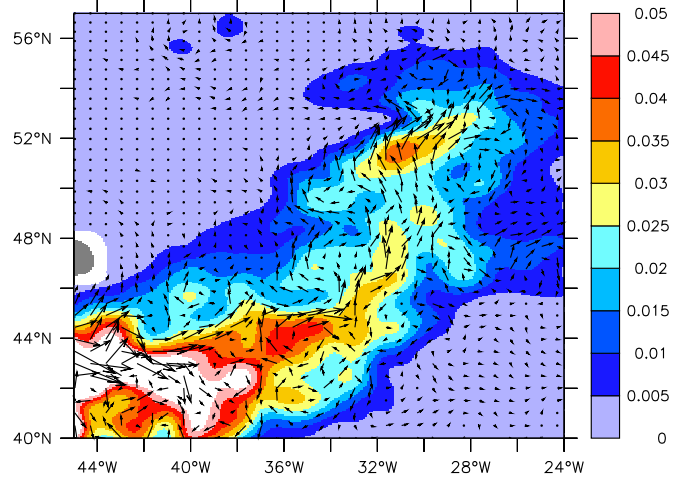


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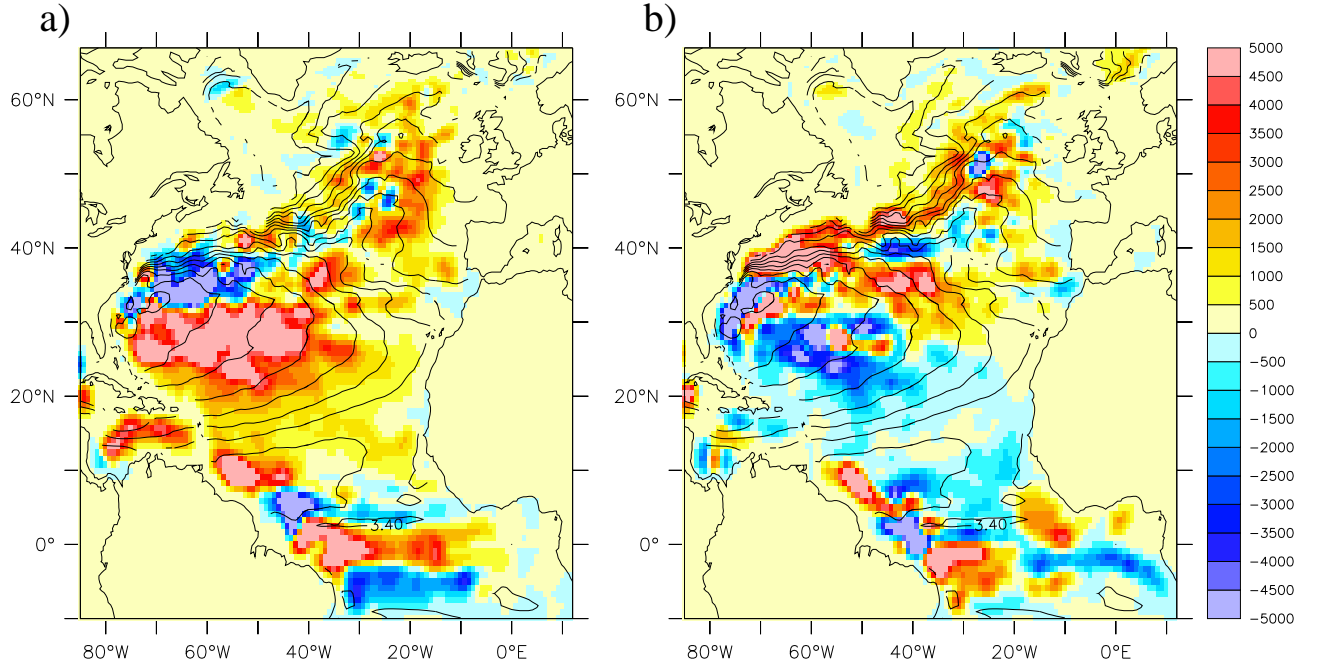


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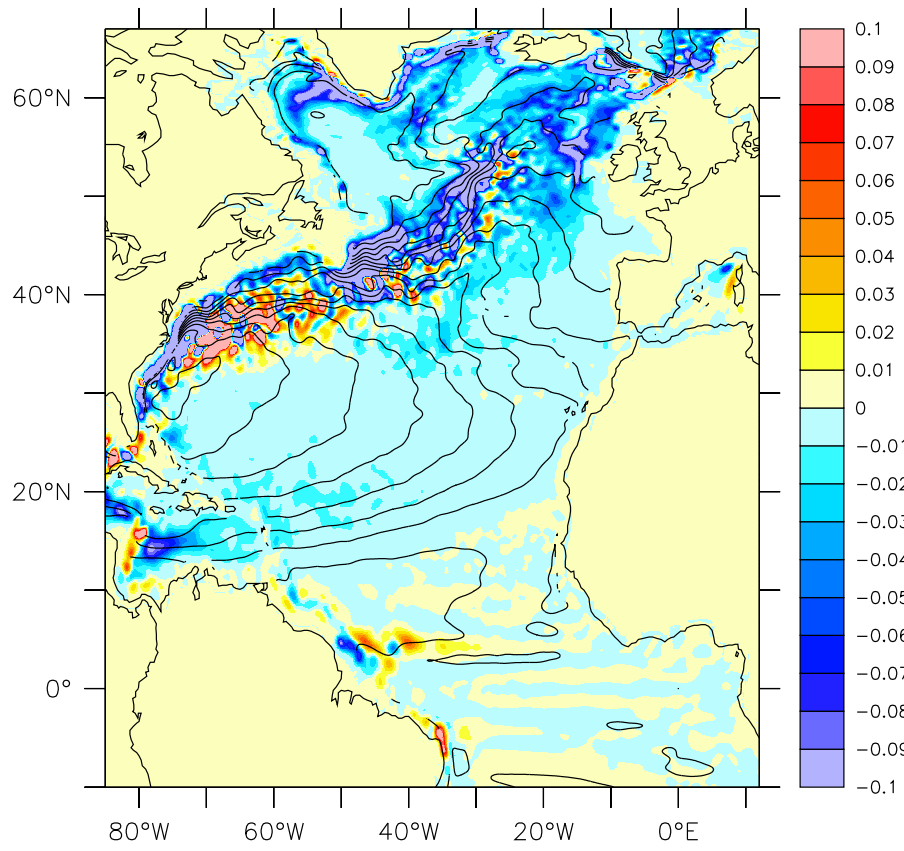


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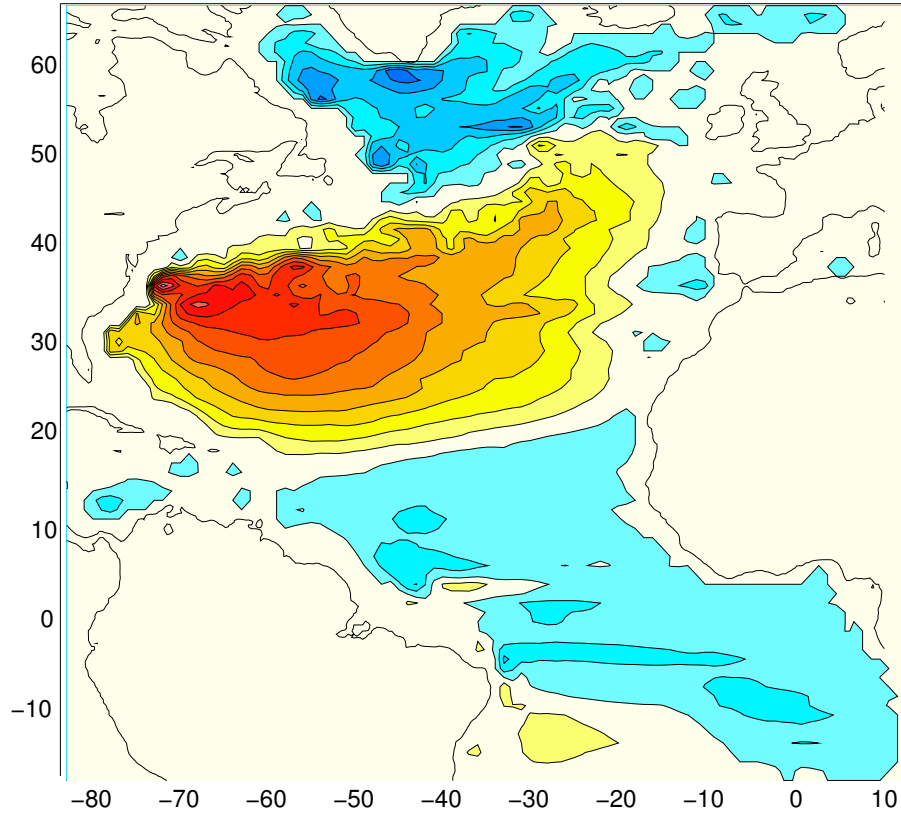


Figure 5: The streamfunction Ψ_m for the mean horizontal flow in 300m depth in m^2/s estimated from $\nabla_h^2 \Psi_m = \nabla_h \cdot \bar{\mathbf{u}}_h$. Contour interval is 5000 m^2/s .

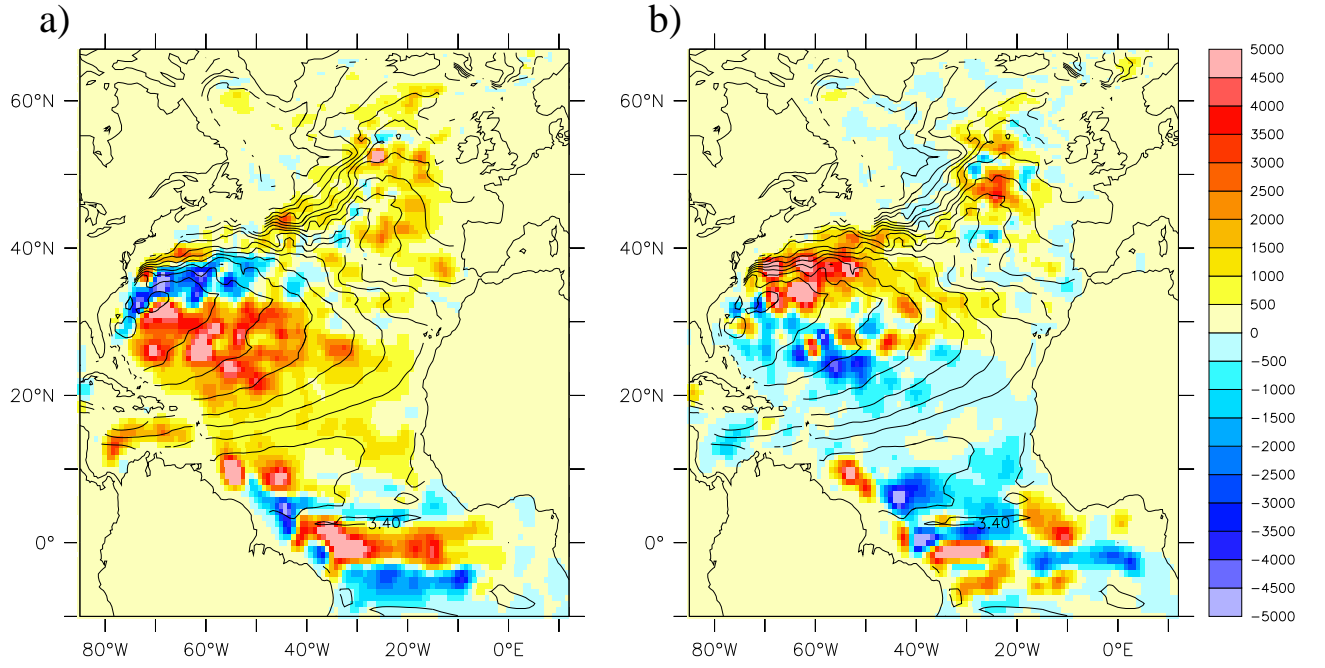


Figure 6: κ_{ms} (a) and ν_{ms} (b) in m^2/s in 300m depth estimated from horizontal eddy fluxes from which a rotational flux following Marshall and Shutts (1981) has been removed. Data are computed from 5 year averages and $\overline{\mathbf{u}'_h \mathbf{b}'}$ and \bar{b} are spatially averaged over 15 grid points. Also shown are contours of the mean buoyancy \bar{b} .

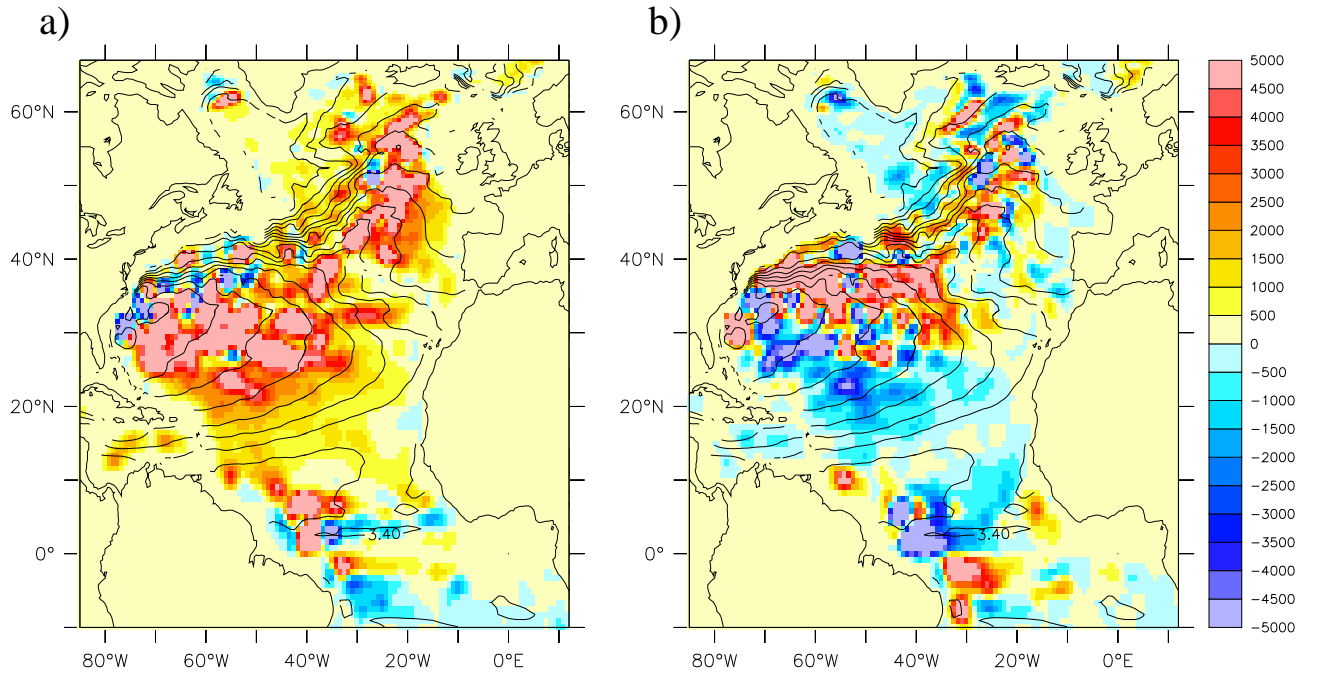


Figure 7: κ_{mg} (a) and ν_{mg} (b) in m^2/s in 300m depth estimated from eddy fluxes from which a rotational flux following Medvedev and Greatbatch (2004) has been removed. Data are computed from 5 year averages and $\overline{\mathbf{u}'_h \mathbf{b}'}$ and \bar{b} are spatially averaged over 15 grid points. Also shown are contours of the mean buoyancy \bar{b} .

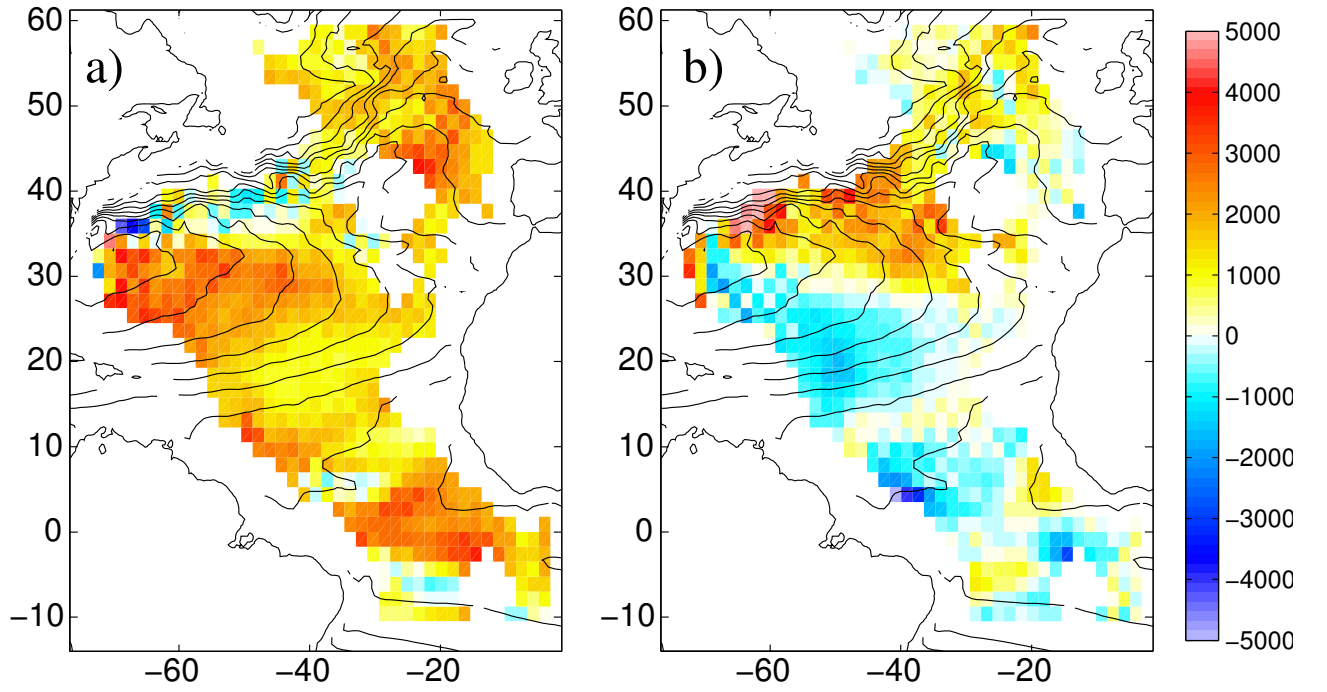


Figure 8: κ (a) and ν (b) in m^2/s in 300m depth estimated from eddy fluxes from which an optimized rotational flux has been removed (exp. MINI-I, see text for details). Also shown are contours of the mean buoyancy \bar{b} .

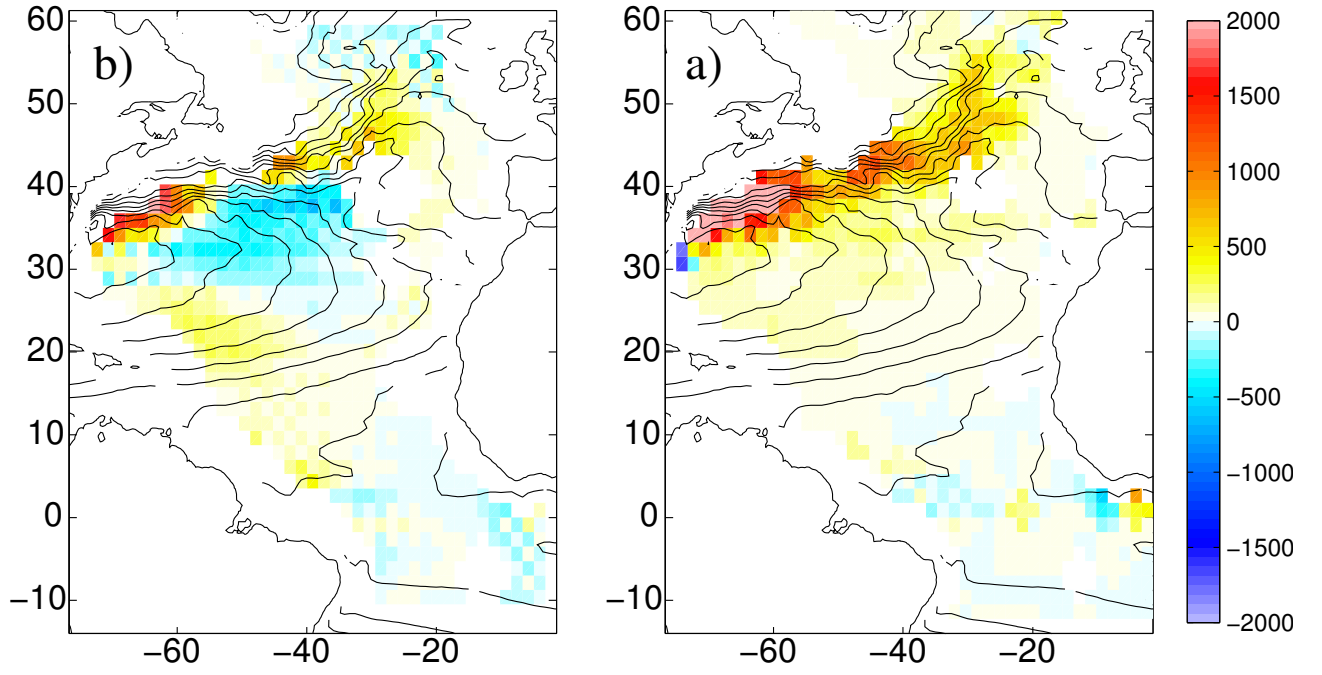


Figure 9: The optimal rotational gauge potential θ_3 (a) and $-\gamma$ (b) in m^3/s^3 in 300m depth (exp. MINI-I, see text for details). Also shown are contours of the mean buoyancy \bar{b} .

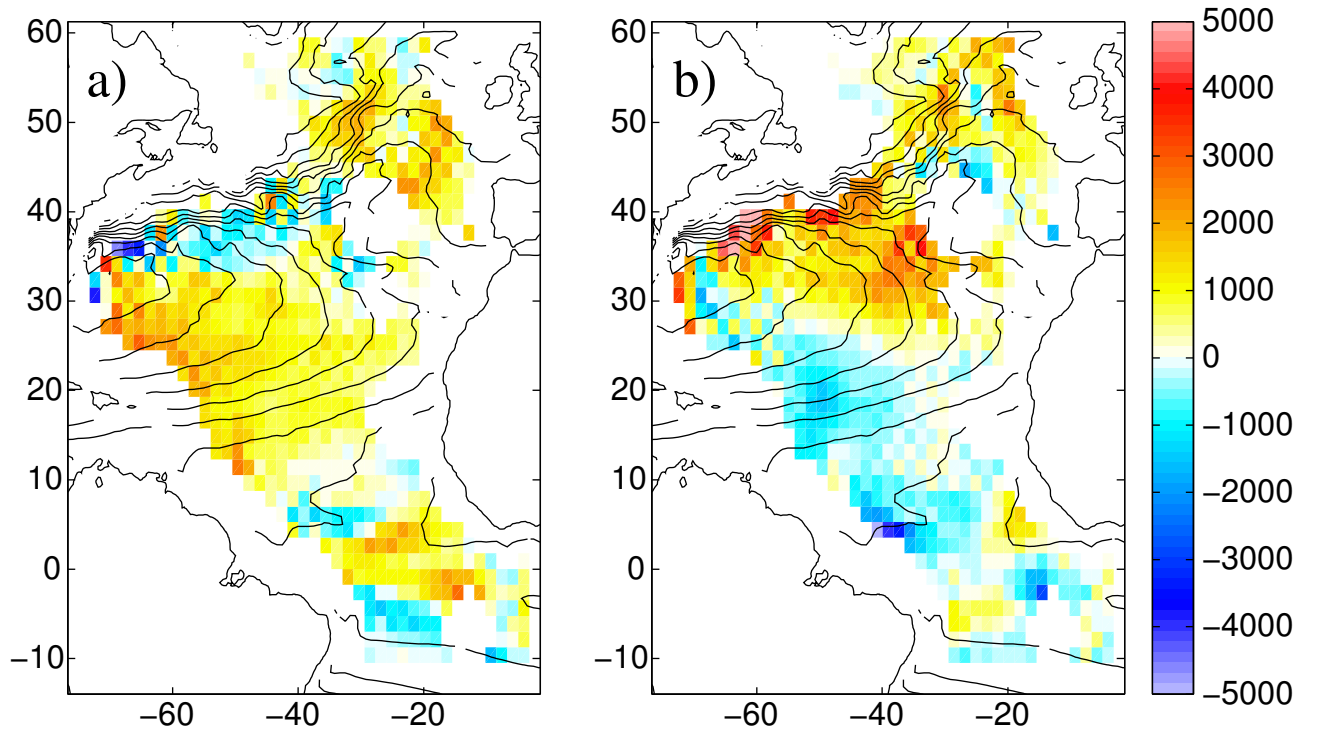


Figure 10: κ (a) and ν (b) in m^2/s in 300m depth estimated from eddy fluxes from which an optimized rotational flux has been removed (exp. MINI-II, see text for details). Also shown are contours of the mean buoyancy \bar{b} .