Numerical experiments on double-diffusive intrusions in the ocean and their relation to laboratory experiments

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Abstract

Numerical experiments on double-diffusive intrusions are reviewed briefly. Though the number of studies is very limited at present, they have undoubtedly an advantage that a heat-salt system can be studied without undesired heat loss from the boundaries. Several possibilities for future numerical experiments are summarized.

Keywords

Thermohaline interleaving, heated sidewall intrusion, double-diffusive gravity current.

Contents

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1. Introduction

Double-diffusive intrusion or interleaving processes are essentially produced by horizontal contrast in temperature and salinity along density surfaces. Because micro-scale processes associated with double-diffusive convection may cause horizontal intrusions that extend over 10 to 100km, it seems to be important to parameterize this process in future numerical modeling of global circulation.

In his pioneering work, Stern (1967) considered intrusions in an infinitely wide front by parameterizing the vertical transports of salt and heat by salt-finger convection. This is now called Stern's parameterization. He showed that warm/salty intrusions rise and cold/fresh ones sink across isopycnals. Motivated by his analysis, double-diffusive intrusions have been investigated both theoretically and experimentally, and the results have been applied to explain the behavior of realistic oceanic intrusion. Previous experimental studies are reviewed by Ruddick (2003), theoretical studies by Ruddick & Kerr (2003) and observational studies by Ruddick & Richards (2003) in this volume.

Numerical experiments on double-diffusive intrusions are few to date, but have certainly some advantages over laboratory experiments; One is control of heat loss. When we deal with the oceanographically relevant heat-salt system in laboratory, heat loss from boundaries often prevents quantitative study. Another advantage is studying the flow field for a relatively wide range of external parameters.

Zhurbas, Kuzmina & Kulsha (1987) first attacked this problem in the simplest case where basic stratification is favorable to onset of salt-finger convection, and tried to simulate the evolution of thermohaline step structure. Walsh & Ruddick (1998) and Merryfield (2000) investigated nonlinear behavior of intrusions in a wide front using a more sophisticated model, and tried to clarify the mechanism that governs the equilibration of intrusions and the formation of thermohaline step structure.

Some other numerical models considered intrusions produced by sidewall heating or cooling of a salt stratified tank (Wirtz, Briggs & Chen, 1972; Chen, 1974; Schladow, Thomasa & Koseff, 1992; Kranenborg & Dijkstra, 1995; Kranenborg & Dijkstra, 1998; and Dijkstra & Kranenborg, 1998). At first glance, these experiments have no direct relation to oceanic double-diffusive intrusions. But, as pointed out by Ruddick & Kerr (2003) in this volume, this approach has some advantages:

1) Avoids using Stern’s parameterization, which is based on laboratory determined flux law that is not completely understood at present.
2) Such experiment is useful for some laboratory experiments where salt-fingers are either not well developed or arise as a secondary instability.
3) Numerical experiments conducted with molecular diffusivities should exactly simulate the laboratory scale experimental results.

The other type of numerical experiments was done by Yoshida & Nagashima (1999) on double-diffusive gravity currents by parameterizing the vertical heat and salt flux by laboratory flux laws. They tried to simulate the lock-exchange flow when relatively cold and fresh water (river runoff) is released on to the warm and salty water (coastal water). Their results are compared with previous laboratory experiments (Thangam & Chen, 1981; Maxworthy, 1983; Yoshida, Nagashima & Ma, 1987). It is noted that McDougall (1985) proposed a frictionless model of surface double-diffusive gravity current. He concluded that the entrainment process reduces the effect of double-diffusion.

The purpose of this paper is to summarize previous studies, and to recommend future problems to be solved by numerical simulations. Therefore, the details of the numerical experiments, such as numerical schemes and resolutions can be found in the original papers referenced.

2. Thermohaline interleaving

In case of simulating double-diffusive intrusions or interleavings, an essential point is how to parameterize the vertical diffusion of heat and salt in the presence of alternating salt-finger and diffusive intrusive layers. Zhurbas et al. (1987) simplified the vertical diffusivity of salt K_S in a fluid with density ratio $R_\rho = \alpha T_z / \beta S_z$ as:

$$K_S = \begin{cases} \begin{Bmatrix} K_0 \exp(-\kappa R_\rho), R_\rho \geq 1 \\ 0, \qquad , R_\rho \leq 1 \end{Bmatrix}, \dfrac{\partial \rho}{\partial z} \geq 0 \\ \infty \text{ instantaneous mixing, } \dfrac{\partial \rho}{\partial z} < 0 \end{cases},$$

and, the vertical diffusivity of temperature K_T through density flux ratio $\gamma = \alpha F_T / \beta F_S$ as:

$$K_T = \begin{cases} K_S \dfrac{\gamma}{R_\rho}, & \dfrac{\partial \rho}{\partial z} \geq 0 \\ \infty \text{ instantaneous mixing, } & \dfrac{\partial \rho}{\partial z} < 0 \end{cases}.$$

Here, z is taken positive downward, γ was taken to be 0.56 (Turner, 1967) and K_0 and κ are constants. Zhurbas et al. (1987) neglected the advection term and calculated from an initial state of alternating fingering and diffusive favorable layers, considering only vertical flux convergence due to salt-fingers. They found that at small values of R_ρ (=1.5), the equilibrium solution has a step structure, and when $R\rho$ is large, only a large amplitude temperature and salinity disturbance can grow into a step structure.

Walsh & Ruddick (1998) and Merryfield (2000) investigated the nonlinear behavior of intrusions in a wide front in a more sophisticated manner. They align horizontal coordinate system along the frontal intrusion slope to eliminate the dependence of perturbation fields on horizontal coordinates. They parameterize the diffusivities according to four categories:

(1) Finger dominant regime, $K_s^{sf} = R_\rho^{-2}$ for Walsh & Ruddick (1998) and $K_s^{sf} = 0.17(1-\tau R_\rho)/(R_\rho - \gamma)$ for Merryfield (2000) based on numerical simulations of salt-fingers by Merryfield & Grinder (unpublished). Here, τ is the Lewis number defined as $\tau = k_S / k_T$.

(2) Diffusive convection dominant regime, $K_s^{diff} = R_\rho^{-3}$ for both experiments.

(3) Stable stratification regime, and

(4) Unstable stratification regime

Both studies added a constant background turbulent diffusivity, K_{turb} (equal for T and S), which dominated in the case of stable stratification and was present in all other cases. This allowed a smooth transition from double-diffusive to stable stratification regimes, prevented the formation of unresolved sharp interfaces, and allowed the effective flux ratio to vary with depth, a key factor in attaining a steady state. In the case of unstable stratification, a larger turbulent diffusivity, K_{conv}, was introduced. The relative values of the double-diffusive, turbulent, and convective diffusivities are arbitrary. Walsh & Ruddick (1998) showed that an infinitesimal intrusion grows to approach an equilibrium state in which frictional forces balance buoyancy forces, the salt/heat flux divergence ratio matches that demanded by the intrusion slope, and the statically unstable convecting layers between salt-finger and diffusive layers are well reproduced (Figure 1). Merryfield (2000) showed that there exists a threshold density ratio value R_ρ^{Stair}. When the density ratio is sufficiently small or the level of background turbulence K_{turb} is sufficiently small, numerical solutions equilibrate as a staircase (Figure 2). For larger R_ρ or K_{turb}, the solution shows the development of intrusive layers.

3. Heated sidewall intrusions

Laboratory experiments on intrusions produced by sidewall heating or cooling a stratified tank fall into two experimental configurations:heating a stable salinity stratified tank (singly stratified case, Thorpe, Hutt & Soulsby, 1969; Chen, Briggs & Wirtz, 1971; Huppert & Turner, 1980; Narusawa & Suzukawa, 1981; Chereskin & Linden, 1986; Tanny & Tsinober, 1988; Bergman & Ungan, 1988) and heating a stably stratified tank having stable salinity and unstable temperature stratifications (Jeevaraj & Imberger, 1991; Schladow et al., 1992)). In both cases, intrusive motions are induced by the instability of fluid parcels near the sidewall. Ruddick (2003) reviews these laboratory experiments in this volume. From these experiments, major questions about the intrusions are:

(1) What are criteria for onset of intrusive layers, in terms of non-dimensional number such as the Rayleigh number (see below this text)?

(2) What are the effects of tank width?

(3) What are the characteristics of merging processes of layers?

(4) What determines the dependence of nose velocities?

(5) What are the role of unstable temperature gradient and the mechanism of self-propagation of intrusions?

Wirtz et al. (1972) investigated flow patterns for the singly stratified case in a finite and an infinite (periodic horizontal boundary conditions) box, and found that at large Rayleigh numbers cell structures appeared at the top and bottom boundaries in a finite box model with additional cells formed successively inward. This formation of the layering is in agreement with experimental results of Mendenhall & Mason (1932). In an infinite box model, successive layer formations are well reproduced, and the thickness of the layer approximately coincided with length scale.

Chen (1974) extended Wirtz et al. (1972)'s treatment to investigate the effects of tank width on the onset of cellular motion, and the dependence of cell layer thickness on the Rayleigh number. He found that the width of tank has little effect on cell growth if the tank width is sufficiently larger than the theoretically predicted typical length scale, and that layer thickness decreases slightly with increasing Rayleigh number.

In addition to the Rayleigh number criterion, Schladow et al. (1992) use $Tu = \dfrac{\beta \partial S / \partial z}{\alpha \partial T / \partial z}$ as an index for gravitational stability and the strength of double-diffusion, and introduce $R_I = -\dfrac{(\alpha q / \kappa_T)}{\alpha \partial T_0 / \partial z - \beta \partial S_0 / \partial z}$ as a lateral stability parameter for doubly stratified (diffusive sense) and constant heat flux configurations. Here, α is the expansion coefficient for the faster diffusing property (T) and β is that for the slower diffusing property (S), q is the constant heat flux applied at the sidewall, and the suffix 0 represents the basic stratification (stable salinity and unstable temperature stratifications). Generally, large R_I indicates strong lateral forcing and weak gravitational stability, and large Tu indicates weak double-diffusive effects and high gravitational stability. They conducted numerical experiments in which flow was generated under low R_I and large Tu. Their experimental domain was restricted to a small size (2.91cm (vertical)$\times$8cm(horizontal)) to observe the early stages of layer formation. They resolve up- and down-going salt-finger structure having scale less than 1mm, and obtained a vertical length scale that coincided with the laboratory results. However, their calculation was limited to the case of highly stable stratification, and both self-propagation and merging mechanisms were not fully explained.

Kranenborg & Dijkstra conducted series of experiments to examine bifurcation characteristics of the equilibrium solutions (Kranenborg & Dijkstra, 1995), self-propagating motion (Dijkstra & Kranenborg, 1998) and merging (Kranenborg & Dijkstra, 1998). They explained that if the whole system is strongly unstable (Tu is sufficiently small), strong upward transport of salt occurred, and the excess salt accumulated in the top layer. This accumulation of warm and salty water on top causes salt-fingering. After sidewall heating is turned off, temperature diffuses faster than salt, and the horizontal temperature difference soon disappeared, while the horizontal salinity difference remained. As a result, a net horizontal density gradient mainly due to horizontal salinity difference is established, and this density gradient causes self-propagation of the intrusion.

The process of layer merging is examined by Kranenborg & Dijkstra (1998). They showed that when one intrusive layer is overtaken by the layers above and below, the velocity shear between the interfaces increases to become unstable (local Richardson number decreases). The overtaken layer is then mixed into the layers above and below. They also obtained an expression for the vertical profile of the horizontal velocity, but did not confirm 1/4 power dependence of velocity on time obtained by Jeevaraj & Imberger (1991).

4. Double-diffusive gravity current

Two kinds of numerical calculation on double-diffusive gravity currents were carried out by Yoshida & Nagashima (1999). For the case of lock exchange flow where salt-finger convection prevails, the resulting interface is not horizontal, but inclines upwards towards the heavier fluid (Fig.3, left hand side). When the density difference between the lock gate due to salt ($\beta \Delta S$) is small, resulting currents behave as a single component lock exchange flow (Fig. 3, right hand side). But when $\beta \Delta S$ is large, the interfacial slope becomes conspicuous. Furthermore, a gravity current is induced even if the initial density anomaly is exactly equal to zero. These results compared well with the laboratory results of Yoshida et al. (1987). It is found that the phenomena are governed by the combination of three parameters. One is the turbulent Prandtl number (ε); when ε is large, the density current is suppressed. A second parameter is the turbulent Rayleigh number (Ra); when Ra is large, induced convection becomes vigorous and the resulting density current becomes strong. The last parameter is the Turner number (Tu) defined by $Tu = \beta\Delta S/\alpha\Delta T$; when Tu is sufficiently small, the activity of double-diffusive convection becomes large. The numerical results also confirmed the relationships for propagation distance L of the current versus time t in various force balance phases obtained experimentally by Maxworthy (1983).

5. Summary

One-dimensional numerical modeling by Zhurbas et al. (1987) and Merryfield (2000) showed fairly well the development of thermohaline staircases at sufficiently small density ratio value. This tendency for staircases to occur at small R_ρ coincides with previous field observations such as C-SALT (Schmitt, Perkins, Boyd & Stalcup, 1987). Walsh & Ruddick (1998) showed the evolution of infinitesimal intrusions to large-amplitude steady state intrusions. However, other dynamical features such as the nose velocity of the interleaving structure could not be investigated in these experiments. The laboratory experiments of Ruddick, Phillips & Turner (1999) found intrusion advance speed U to be proportional to intrusion thickness H and buoyancy frequency N as $U = 0.005NH$. As for sidewall heating problem, Ruddick (2003) suggested that nose velocity should have same tendency. However, laboratory and numerical experiments on double-diffusive gravity currents in the absence of background stratification necessarily showed different behavior of nose velocity. The exact evaluation of nose velocity should be examined in future numerical modeling to estimate effective lateral diffusivity of interleaving. However, in laboratory or numerical tank, the aspect ratio of intrusion is larger than that found in the ocean, and the behavior of such intrusions should also be affected by the presence of end walls. Thus, we must be very careful if we apply lateral velocities or horizontal fluxes from laboratory or numerical results directly to the ocean.

One great advantage of numerical experiments is that the full three-dimensional structure of the flow can be captured. In this context, the effect of rotation on double-diffusive intrusion should be investigated. This laboratory experiment is only done by Chereskin & Linden (1986) for sidewall heating problem, and showed substantial differences in the intrusion layer thickness. Yoshida, Nagashima & Niino (1989) investigated theoretically the behavior of intrusions at a finite front having density compensating temperature and salinity gradients. They showed that the most unstable mode has larger growth rate than that obtained in an infinite front model under the effects of rotation (e.g. Toole & Georgi, 1981). Richards (1991) analyzed theoretically an equatorial intrusion with the effect of equatorial β plane, and obtained a horizontal length scale (100km) that agrees well with the observation. These results indicate the importance of rotation on rather large-scale intrusions, and numerical experiments will help in understanding the mechanism for such long-lived, large-scale intrusions.

Another advantage of numerical experiments is that one can use molecular diffusivities and hence can avoid using flux parameterization or other unknown parameters. However, the large scales of oceanic double-diffusive phenomena means that numerical experiments would need to resolve from scales of perhaps less than mm (to resolve salt diffusion) to hundreds of m scale. The largest numerical domain to date using molecular values is only 20cm$\times$20cm (Kranenborg & Dijkstra, 1995, Dijkstra &

Kranenborg, 1998). If we use laboratory flux laws, this allows the minimum resolved scale to become much larger, but the numerical domain of Yoshida & Nagashima (1999) was only slightly extended to 50cm$\times$6cm. In addition, the results strongly depend on the unknown flux laws, so we can't compare to the ocean with any confidence. We conclude that devising and testing good vertical flux parameterizations is necessary before we can make progress in numerical modeling.

Acknowledgements

The authors expresses their sincere thanks to Yuli Chashechkin and Joe Fernando who co-chairs our SCOR working group (WG 108). Thanks are extended to Ann Gargett, Oliver Kerr, Eric Kunze, Barry Ruddick, for their valuable comments. Their efforts greatly improved this manuscript. Two anonymous reviewers are also greatly acknowledged.

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Figure Captions

Figure 1 Evolution of alternative fingering, convecting (statically unstable) and diffusive layers. (Walsh & Ruddick, 1998). (Reproduced by permission of American Meteorological Society).

Figure 2 Equilibria of intrusions under C-SALT conditions. Salt-finger flux is parameterized with flux law obtained by numerical calculation by Merryfield & Grinder (unpublished). Density ratio R_ρ is taken as the abscissa and background turbulent eddy diffusivity K_{turb} is taken as the ordinate. The heavy solid line shows R_ρ^{Stair}. Inserted four panels show the profiles of salinity (solid lines on the left of each panel), temperature (dashed lines), and horizontal velocity (on the right of each panel). On the left side of R_ρ^{Stair} line, where, $R_\rho < R_\rho^{Stair}$, thermohaline staircases are seen, whereas, intrusive structures are seen where $R\rho > R_\rho^{Stair}$. This figure also shows that if R_ρ approaches R_ρ^{Stair}, K_{turb} must become small to form step structures. (Merryfield, 2000). (Reproduced by permission of American Meteorological Society).

Figure 3 (Left) Lock exchange flow with the effect of double-diffusion, (a) Stream function, (b) temperature, (c) salinity and (d) density. (Right) Lock exchange flow without double-diffusion, (e) Stream function, (f) temperature and (g) and density. Note the change of interfacial slope in the double-diffusive case. (Yoshida & Nagashima, 1999)

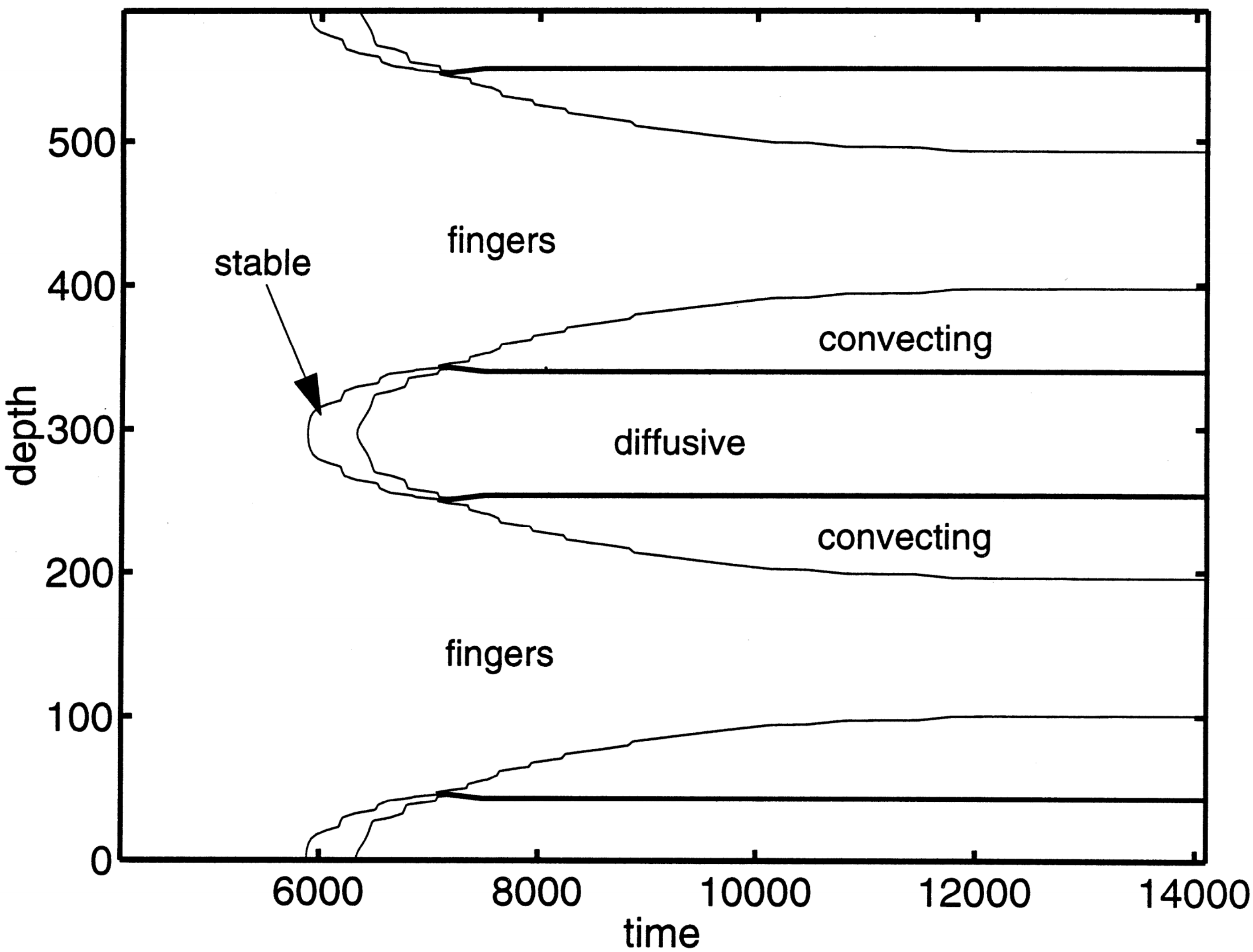

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Figure 1

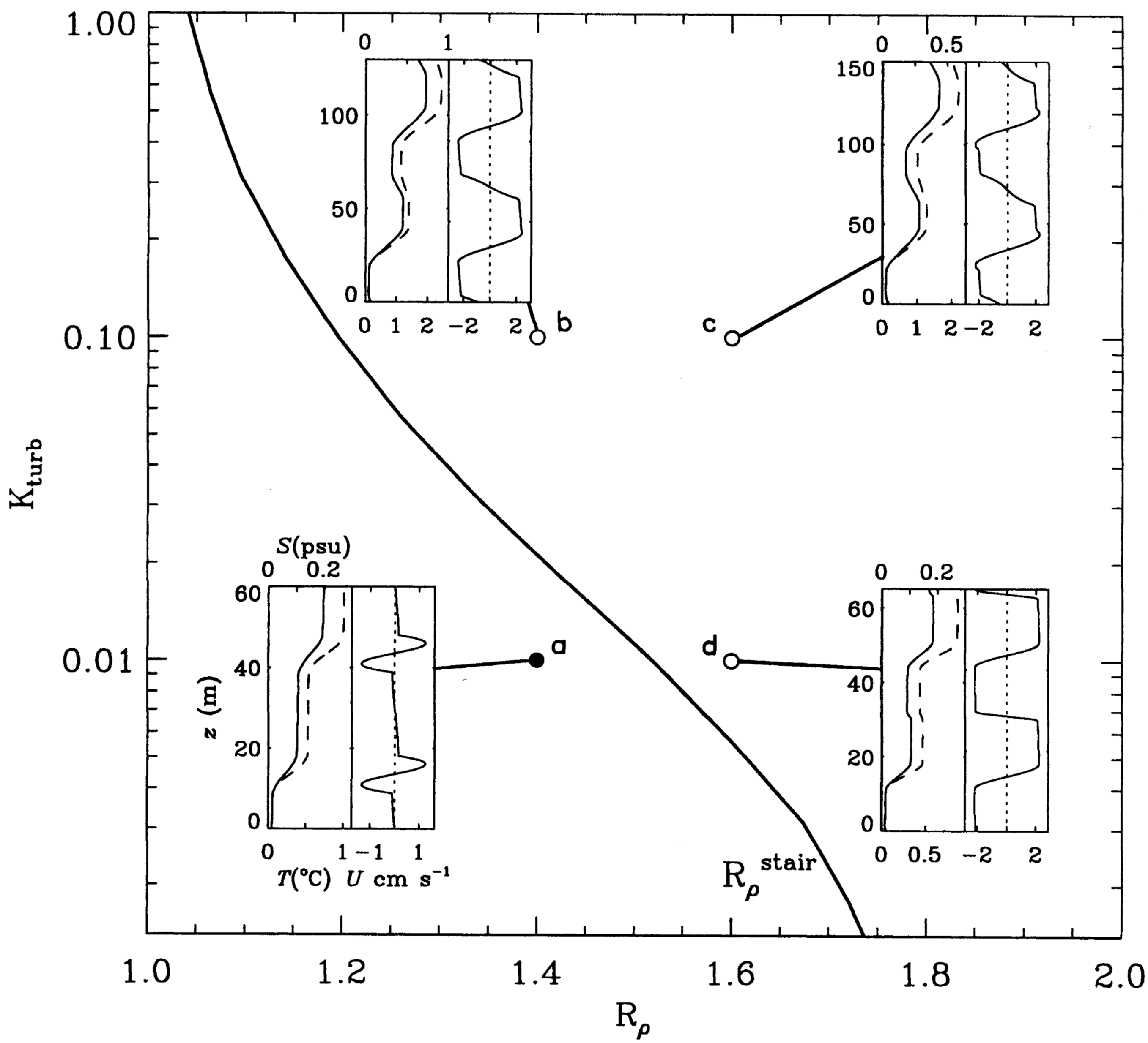

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Figure 2

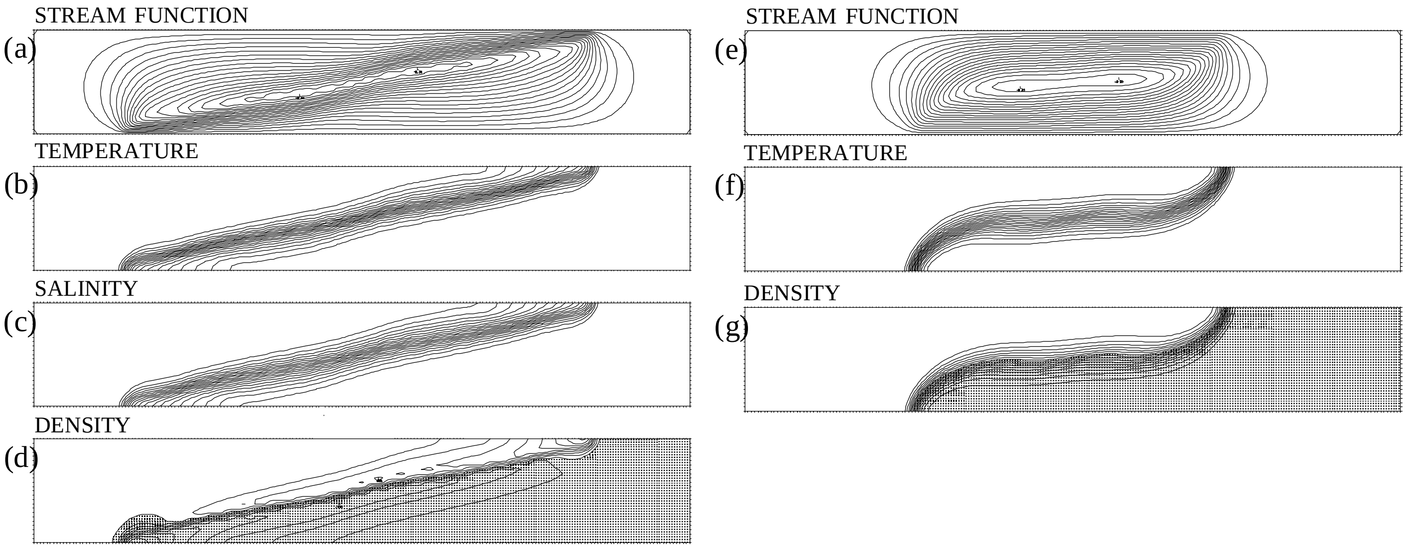
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Figure 3