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VESSEL COLLISIONS WITH WHALES: THE PROBABILITY OF LETHAL INJURY BASED ON VESSEL SPEED

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ABSTRACT

Historical records demonstrate that the most numerous, per capita, ocean-going-vessel strikes recorded among large-whale species accrue to the North Atlantic right whale (*Eubalaena glacialis*). As vessel speed restrictions are being considered to reduce the likelihood and severity of vessel collisions with right whales, we present an analysis of the published historical records of vessels striking large whales. We examine the influence of vessel speed in contributing to either a lethal injury (defined as killed or severely injured) or a nonlethal injury (defined as minor or no apparent injury) to a large whale when struck. A logistic regression model fitted to the observations, and consistent with a bootstrap model, demonstrates that the greatest rate of change in the probability of a lethal injury (P_{lethal}) to a large whale occurs between vessel speeds of 8.6 and 15 knots where P_{lethal} increases from 0.21 to 0.79. The probability of a lethal injury drops below 0.5 at 11.8 knots. Above 15 knots, P_{lethal} asymptotically approaches 1. The uncertainties in the logistic regression estimates are relatively large at relatively low speeds (*e.g.*, at 8 knots the probability is 0.17 with a 95% CI of 0.03–0.6). The results we provide can be used to assess the utility of vessel speed limits that are being considered to reduce the lethality of vessels striking the critically endangered North Atlantic right whale and other large whales that are frequent victims of vessel strikes.

Key words: vessel strike, vessel speed, lethal injury, whales, right whale, probability, logistic regression, bootstrap.

Recently compiled historical (1885 through 2002) records of vessels striking large whales worldwide ($n = 294$; Laist *et al.* 2001, Jensen and Silber 2003) reveal the most frequently reported victims of vessel strikes to be fin (*Balaenoptera physalus*), humpback (*Megaptera novaeangliae*), North Atlantic (NA) right (*Eubalaena glacialis*), gray (*Eschrichtius robustus*), and several other large whales (Fig. 1). On a per-capita basis using contemporary worldwide population-size estimates (Aguilar 2002, Clapham 2002, Ford 2002, Horwood 2002, Jones and Swartz 2002, Kato 2002, Kenney 2002,

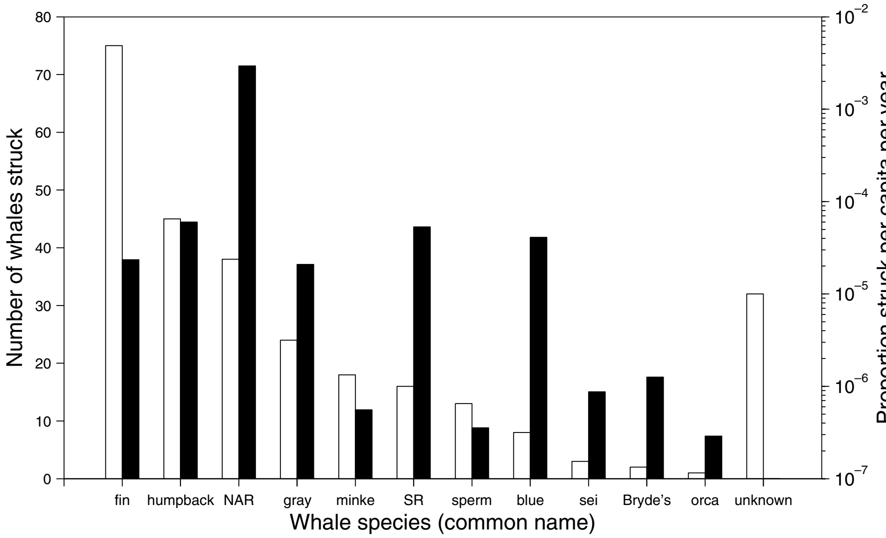

Figure 1. Frequency histograms of worldwide documented (Laist *et al.* 2001, Jensen and Silber 2003) numbers of large whales, including North Atlantic (NAR) and southern (SR) right whales, reported struck by vessels for the period 1960 though 2002 only (open bars), and the same data presented as a temporally adjusted per capita rate (solid bars; $\log_{10}$ scale) using contemporary population size estimates for each species (Aguilar 2002, Clapham 2002, Ford 2002, Horwood 2002, Jones and Swartz 2002, Kato 2002, Kenney 2002, Perrin and Brownell 2002, Sears 2002, Whitehead 2002) where the proportion struck per capita per year = (number of species-specific whales struck/contemporary species-specific population size)/43 years. Where a range in population size was provided, we use the midpoint of the range.

Perrin and Brownell 2002, Sears 2002, Whitehead 2002), and relative to all other large whales reported struck over the period 1960–2002 inclusive (n = 275), the NA right whale is two orders of magnitude more prevalent as victim (Fig. 1). These statistics suggest that relative to other large whales, NA right whales are more prone to being struck by vessels.

Following the U.S. National Oceanic and Atmospheric Administration (NOAA) advance notice of proposed rulemaking (Federal Register (USA) 2004) for right whale ship-strike reduction, Kraus *et al.* (2005) called for emergency measures to reduce ocean-going vessel speeds in east-coast regions of the United States and thereby to reduce vessel-related NA right whale mortality. The call for emergency measures rested on arguments that (1) the NA right whale is the most endangered species of baleen whale (Kraus *et al.* 2001); (2) the population size is diminishing (Fujiwara and Caswell 2001); (3) species extinction is expected within ~200 years unless human-induced kills are reduced (Caswell *et al.* 1999); (4) of all documented kills, most are attributable to vessel-strike (Knowlton and Kraus 2001); and (5) contemporary vessel-kill rates remain high (Kraus *et al.* 2005). Subsequently, and in an attempt to reduce mortalities due to vessel strikes, the NOAA proposed rule (Federal Register (USA) 2006) aims to "impose vessel speed restrictions of 10 knots or less" in "certain areas and at certain times of the year, or under certain conditions," and "also invites comments on vessel speed restrictions of 12 knots or less, and 14 knots or less."

The above observations, arguments, and proposals led us to estimate the probability of a lethal injury (*i.e.*, killed or severely injured) to a large whale as a function of vessel speed at the time of the vessel–whale collision. We report statistically determined estimates of the probability of a lethal injury and their associated 95% confidence intervals (CIs) based on vessel speed and offer the estimates as a first step toward assessing the utility of vessel speed restrictions in areas where vessels are likely to encounter whales.

METHODS

We use the only published sources detailing the historical record of vessels striking large whales ($n = 294$; Laist *et al*. 2001, Jensen and Silber 2003) where the records ($n = 47$) jointly provide the vessel speed estimate and the severity of injury to the stricken whale. Laist *et al*. (2001) describe four injury classes: killed (carcass observed); severe (bleeding wounds and/or blood in the water); minor (visible nonbleeding wound, signs of distress, no report of blood); none apparent (resighted, no visible wound or distress, animal resumed prestrike activity); and a 5th unknown-injury class (animal not observed again and no report of blood). Jensen and Silber (2003) assess injury differently, though their descriptions allowed us to classify according to the four injury classes of Laist *et al*. (2001). Those data where speed was known and injury was unknown are excluded. "Unknown" species are included where speed was known. Apart from the unknown species, all but one record (*Orcinus orca*, retained) involved large whale species (Fig. 1). We use knots as the unit of speed as it is the nautical convention. Vessel speed is classified in two-knot intervals for all analyses except in the chi-square tests described below. If vessel speed was reported as a range, the midpoint is used. One case reported <10 knots for a vessel accelerating at the time of strike and a speed of 10 knots is assumed for the analyses.

The few data detailing vessel collisions with right whales require us to assume that the other large whales, primarily baleen whale species (Fig. 1), serve as suitable proxies, at least from a body-mass perspective. This assumption is justified by the average mass at maximum length relation provided by Trites and Pauly (1998) that shows one relation applies to all mysticetes and sperm whales (*Physeter macrocephalus*) with a mid point mass of 42.5×10^3 kg. Additionally, the average mass across all species, excluding *Eubalaena sp*. and *O. orca,* based on data provided by Lockyer (1976), is 39×10^3 kg ($n = 219$, CV = 84%), and the mean of the species-specific means is 31×10^3 kg. These estimates above are broadly consistent with the 39×10^3 kg estimate for a 20 year-old right whale (Moore *et al*. 2004).

Chi-square tests are used to assess the independence of vessel speed and the severity of injury according to the four injury-classes of Laist *et al*. (2001) above. We employ the simple logistic regression model, $P_{\text{lethal}} = \frac{1}{1+\exp^{-(\beta_0+\beta_1 \text{speed})}}$, (*e.g.*, Myers *et al*. 2002) using mid point speeds among the two-knot speed classes, the proportion of whales suffering either "nonlethal" or "lethal" injury, and maximum likelihood estimation to determine the parameters and the CIs around model estimates. We define nonlethal as the sum of the minor and none-apparent injury classes above, and lethal as the sum of the killed and severe injury classes above. In the latter case, we explicitly assume a severely injured whale ultimately succumbs to the injury. This assumption has some merit for a number of reasons (1) other evidence of vessel strikes, such as scars from propeller wounds on live animals, has a low incidence of reporting (7%) and is interpreted as indicating such strikes are deadly to NA right whales (Kraus 1990);

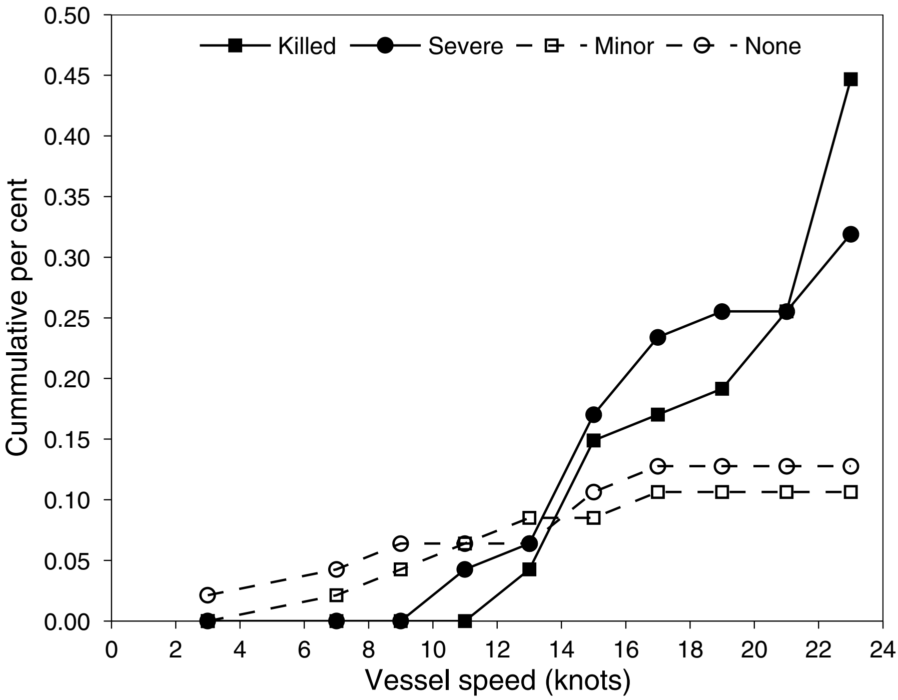

Figure 2. Cumulative per cent increase in each of four whale-injury classes as a function of the midpoint of the two-knot vessel-speed classes illustrating how the killed and severe injury classes increase similarly and in parallel, as do the minor and none apparent injury classes.

(2) of the documented vessel strikes in the NA right whale population, 1/2 of known propeller injuries proved fatal (Knowlton and Kraus 2001); (3) blunt trauma that is consistent with vessel strike is not externally obvious and frequently results in death (Wiley *et al.* 1995, Best *et al.* 2001, Moore *et al.* 2004); and (4) the cumulative percent of the killed and severe-injury classes in the data we examine increase similarly and in parallel with speed as do the cumulative percent minor and none-apparent injury classes, though the latter at a lower level (Fig. 2). The results of the logistic regression are used to draw inferences based on its inflection as well as on the two inflections of the first derivative of the functional relation.

The reliability of the data and the simple logistic regression model are examined using a bootstrap technique computed using "R" (R Development Core Team 2005) by resampling the data, with replacement, 1,000 times and by fitting the logistic to the resultant predicted probability distributions (based on nonlinear least squares estimation) across speed classes.

Results

Speed and injury are not independent (6 df, $P = 0.014$) when vessel speed is categorized across three 8-knot speed intervals: low ($0 \leq$ knots ≤ 8), moderate ($8 <$ knots ≤ 16), and high (>16 knots); that is, as speed increases the severity of injury

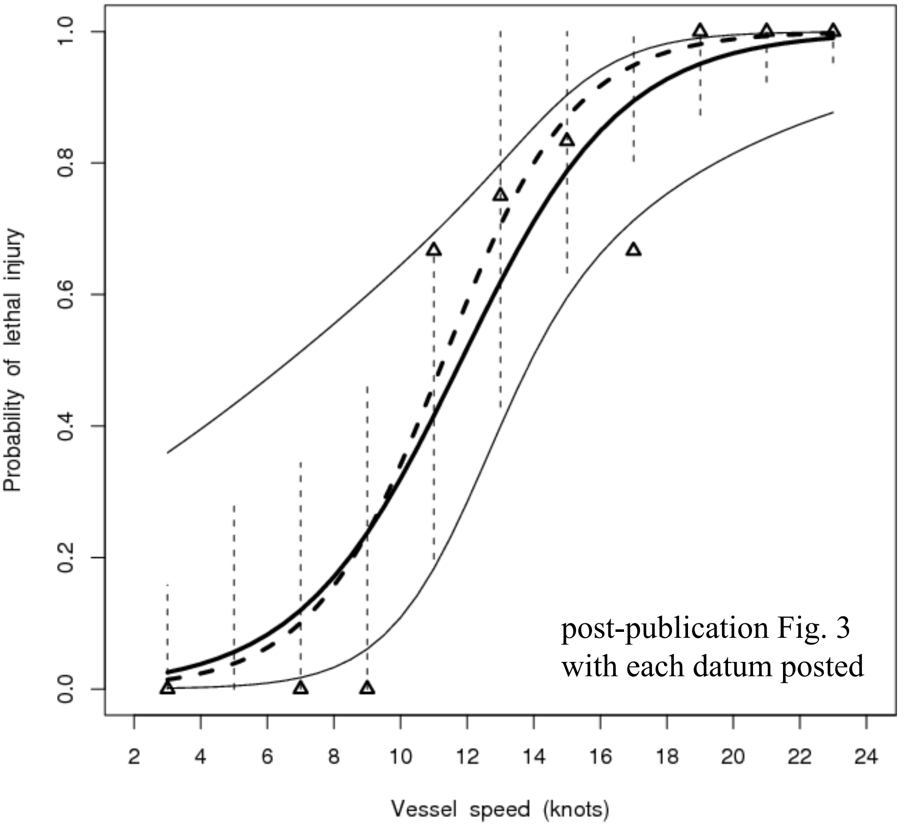

Figure 3. Probability of a lethal injury resulting from a vessel strike to a large whale as a function of vessel speed based on the simple logistic regression (solid heavy line) and 95% CI (solid thin lines) and the logistic fitted to the bootstrapped predicted probability distributions (heavy dashed line) and 95% CI for each distribution (vertical dashed line) where each datum (Δ) is the proportion of whales killed or severely injured (*i.e.*, lethal injury) when struck by a vessel navigating within a given two-knot speed class. There are no data in the 4–6 knot speed class.

increases. The same test based on four-speed classes incrementing at six knots and three-speed classes incrementing at 10 knots, and assessed against the four severities-of-injury, leads to the same conclusion (9 df, $P = 0.0007$ and 6 df, $P = 0.0001$, respectively).

The probability of a lethal injury (Fig. 3) as a function of vessel speed (knots) is determined as: $P_{\text{lethal}} = \frac{1}{1+\exp^{-(-4.89+0.41\text{speed})}}$. Wald's chi-square shows both β_0 and β_1 as different from zero ($P = 0.013$ and 0.003, respectively), and the overall model is significant ($P < 0.001$) according to a likelihood ratio test. The logistic fitted to the bootstrapped probability distributions has similar parameter estimates: $\beta_0 = -5.76$ and $\beta_1 = 0.51$.

The simple logistic regression model (Fig. 3) shows that the greatest rate of change in the probability of a lethal injury to a large whale occurs between 8.6 knots

Table 1. The odds ratio and associated lower and upper 95% confidence limit of a lethal injury to a large whale occurring at a given vessel-speed increment.

Speed increment (knots)	Odds ratio	Lower 95% limit	Upper 95% limit
1	1.51	1.15	1.99
2	2.29	1.32	3.94
3	3.45	1.52	7.83
4	5.22	1.75	15.5
5	7.89	2.02	30.9

($P_{\text{lethal}} = 0.21$) as defined by the first inflection of the first derivative of the logistic and 15 knots ($P_{\text{lethal}} = 0.79$) as defined by the second inflection of the first derivative. Only at speeds below 11.8 knots (inflection of the logistic) does the probability of a lethal injury drop below 0.5, though the uncertainties around the estimates are large. Above 15 knots P_{lethal} asymptotically approaches 1. The odds ratio, that is the ratio of the odds, $\frac{P_{\text{lethal}}}{1-P_{\text{lethal}}}$, of a lethal injury occurring at a given initial speed relative to the odds at some incremented speed, increases with the magnitude of the speed increment (Table 1). For example, an increase in vessel speed by 1 knot increases the odds of a lethal injury 1.5-fold (95% CI 1.2 2.0) regardless of initial speed. A two-knot increase in speed increases the odds by 2.3-fold (95% CI 1.3 3.9) and a five-knot increase leads to a 7.9-fold (95% CI 2.0 31) increase in the odds of a lethal injury.

The logistic fitted to the bootstrapped (with resampling) predicted probability distributions provides statistically similar results (Fig. 3), and there is no difference in the predicted values derived from the logistic fitted to the bootstrapped probability distributions and those provided by the simple logistic regression model (bootstrapped parameters are well within ± 1 SE of the simple logistic parameter estimates). For this reason the inferences below rely on estimates derived from the simple logistic regression model and the associated 95% CI.

Discussion

The logistic regression model estimates demonstrate that the greatest rate of change in the probability of a lethal injury to a large whale, as a function of vessel speed, occurs between the inflections of the first derivative of the logistic model; that is, between vessel speeds of 8.6 and 15 knots. Across this speed range, the chances of a lethal injury decline from approximately 80% at 15 knots to approximately 20% at 8.6 knots. Notably, it is only at speeds below 11.8 knots that the chances of lethal injury drop below 50% and above 15 knots the chances asymptotically increase toward 100%.

The data used in our analyses are limited and do not incorporate all variables (*e.g.*, species of whale, age, size or mass, and behavior; and vessel type, size or mass, and angle of attack) relevant to vessel–whale collisions. They are, however, the only published data that include vessel-speed observations. Consequently, the CIs are large, particularly at low vessel speeds (<10 knots) where there are few observations. Assuming that the mass of the vessels represented in the data are much greater than the mass of the whales struck, we conclude that vessel speed is sufficient to predict the probability of a lethal injury if a whale is struck, where lethality includes killed or severely injured. This conclusion is not unreasonable, at least within the limits of the two extremes of elastic or perfectly inelastic collisions in one dimension and by assuming that both the mass and speed of the colliding vessel are much greater than the mass and speed of the colliding whale. In such a simplification, it can be shown that it is only the mass of the whale and the speed of the vessel that contribute to the impact forces (see Appendix) and presumably the severity of injury to the whale. Although this simplification ignores the time over which the collision occurs (Δt in Appendix) and how the energy is dissipated during the collision (neither easily determined), it does demonstrate that vessel speed is expected to be a reasonable predictor of lethality—at least as a first approximation. It is notable that the functional forms of the ascending limbs of the logistic models illustrated in Figure 3 are proportional to the square of the vessel speed and thus consistent with expected collision-related kinetic-energy dissipation in the whale.

This study provides insights into the role vessel speed plays in determining the fate of a right whale, or other large whale, if struck. The probability estimates and their associated 95% CIs provide insight into how effective vessel-speed restrictions might be in reducing the severity of vessel-strike injuries. Such restrictions may complement other efforts designed to reduce vessel strikes (Kraus *et al.* 2005). Despite increased awareness of the vessel-strike problem and changes to vessel routing, such as the modified traffic separation scheme in the Bay of Fundy right whale habitat (International Maritime Organisation 2003), there has not been a reduction in the reporting of lethal vessel-strike injuries. There were at least three and possibly four right whale deaths attributed to vessel strikes in the 16 months prior to Kraus *et al.* (2005). It is possible that increased awareness may be responsible for increased reporting. However, if contemporary average vessel speeds of 14–16 knots through two critical right whale habitats (Ward-Geiger *et al.* 2005) are maintained, it is reasonable to expect the probability of lethal vessel-strike injuries to remain in the 0.70–0.85 range based on the simple logistic model (Fig. 3).

One factor our analysis cannot address is the consequence of increased whale exposure to vessels navigating at low speed. Therefore, we briefly explore average vessel–whale encounter probability (P_m) and how it may change as vessel speed decreases. We do this by employing a model, in two dimensions, of a random walk (whale), in the presence of traps (vessels), provided by Gallos and Argyrakis (2001). The probabilities are explored within a specified areal domain, using a vessel frame of reference and a randomly moving whale with a speed that is the sum of the vessel (v_v) and the whale (v_w). In this example, and for simplicity, we assume a square domain of dimension (a) and length (l_v) and beam (b_v) of the vessel and the whale (l_w and b_w). To determine the number of steps in the random walk, we require the time (t_v) for the vessel to transit the domain and the area (c_v) occupied by the transiting vessel within the domain. We approximate that, on average, a vessel transit parallels the edges of the domain; thus, $t_v = \frac{a}{v_v}$. Vessel area within the domain is defined by the number of vessels (N) and their dimensions: $c_v = \frac{N \cdot l_v \cdot b_v}{a^2}$. During the time the vessel transits the domain, the whale will move through an area specified as $A_w = b_w(v_w + v_v)t_v$. The above equations are used to determine the number of steps (S_n) taken by the whale during its random walk: $S_n = \frac{A_w}{l_w b_w} = \frac{a}{l_w}(\frac{v_w}{v_v} + 1)$. There are other means of deriving S_n and in this derivation the whale becomes one-dimensional (l_w). Gallos and Argyrakis (2001) define the average "survival" probability (*i.e.*, no encounter) as $P_s = e^{-\lambda S_n}$, where $\lambda = -log_e(1-c_v)$. Thus, the average probability that the vessel will encounter the whale is, $P_m = 1 - P_s$. We use an example vessel ($l_v = 125$ m, $b_v = 20$ m) and example length ($l_w = 16.5$ m) and swimming speed ($v_w = 1.5$ ms^{-1})

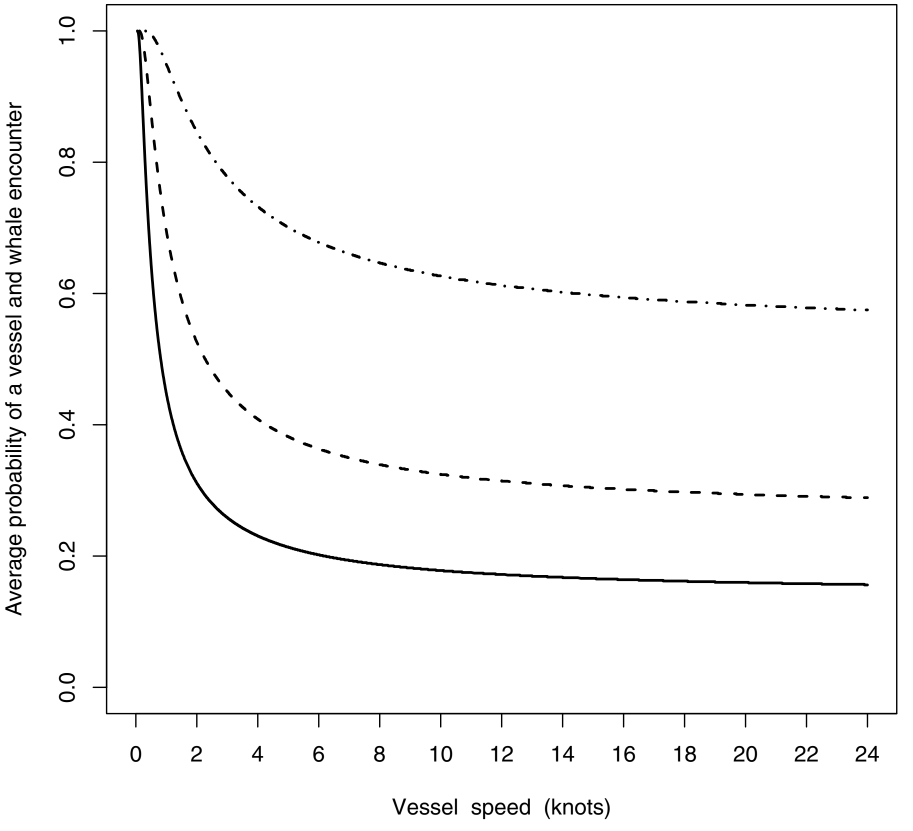

Figure 4. The probability of a vessel and whale encounter, as a function of vessel speed, within a 1 km^2 domain estimated using a random walk model in two dimensions of a 16.5 m whale swimming at 1.5 ms^{-1} in the presence of an example vessel (125 m length and 20 m beam). The lines represent the domain with one whale and one vessel (solid), two vessels (dash), and five vessels (dash dot).

of a whale within a 1 × 1 km domain. Vessel number and vessel speed (in this example vessels have identical dimensions and speed) are varied in the presence of one whale in the domain. Although slow-moving vessels spend more time within the domain than fast-moving vessels, this simple model (Fig. 4) demonstrates that the encounter probability increases slowly as speed decreases from 24 knots or greater and then begins to increase more rapidly as vessel speed continues to decrease toward zero. This model represents an approximation of average encounter probabilities as a function of vessel speed, yet it serves to illustrate that the encounter probability does not increase with decreasing speed as simply as one might expect. Determining such probabilities will be much more complex as the size and shape of the domain (habitat) changes, as the number, sizes, and speeds of vessels and how they transit the domain changes, as well as how the number, sizes, and speeds of whales and how they move in the habitat changes.

Slow-moving vessels may provide opportunity for whales to avoid a collision or for vessel operators to avoid the whales. However, we are unaware of any compelling evidence for either. According to Nowacek *et al*. (2004), NA right whales show neither a behavioral response to the sounds of an approaching vessel nor to actual vessels and suggest that NA right whales may be habituated to vessels noise and ignore it. Southern right whales do not elicit "strong boat-avoidance" behavior (Best *et al*. 2001). Terhune and Verboom (1999) report an adult NA right whale turning into the path of a small motor-vessel and cite Mayo and Marx (1990; though we cannot verify) that on 64 of 138 occasions, NA right whales turned toward a parallel-running small motorized vessel. For a vessel operator to avoid a collision with a whale, the whale must first be detected and the operator must then maneuver to avoid the collision. Large vessels navigating at low speed may not be able to maneuver successfully where success is partially dependent on the operator's ability to predict the movement of the whale once detected. Whale detection is dependent on the surface profile of the whale (right whales have no dorsal fin and thus minimum profile), unpredictable whale behavior, lighting, meteorological conditions (day or night, fog, sea-state, *etc*.), and observer bias (Hain 1997). Laist and Shaw (2006) report that small vessel operators are unable to consistently detect and avoid manatees, and Best *et al*. (2001) report a vessel collision with two or more whales where no avoidance action was taken because the vessel operator anticipated the whales would dive to avoid the vessel.

We cannot dismiss vessel or whale avoidance of a pending collision as explaining the few low-speed collision reports in the data we analyzed. We can suggest that the paucity of low-speed collision reports is related to a paucity of vessels operating at low speed. Our analysis of at-sea vessel speeds, associated with 1989–2002 mandatory ($>$500 gross registered tons) and voluntary vessel reporting in the NW Atlantic, shows 11.5% of the vessels navigating at $\leq$9 knots and 6.2% at $\leq$7 knots (n = 98,562; Eastern Canada Traffic Regulating System, ECAREG, unpublished data). It is also possible that the few reports of vessel collisions with whales prior to 1960 (19 of the 294 records) may be related to (1) lower vessel speeds in earlier decades and associated whale or vessel avoidance, and/or (2) collisions not being reported because of an absence of interest in reporting and/or concern regarding vessel strikes. In the first case, we have little quantitative evidence with which to reject the possibility, although we note that of the nineteen pre-1960 collision reports, only six include a vessel speed at the time of collision, and all six were $\geq$13 knots. Thirteen knots is the contemporary mean vessel speed for the ECAREG data analysis noted earlier, and it is consistent with the 14–16 knot contemporary average speed estimates of Ward-Geiger *et al*. (2005). In the second case, we simply have no evidence to reject, or not, the possibility.

In summary and acknowledging the uncertainties, our analyses provide compelling evidence that as vessel speed falls below 15 knots, there is a substantial decrease in the probability that a vessel strike to a large whale will prove lethal. The estimates we provide can be used to consider the efficacy of vessel speed limits that have been proposed in the United States (Federal Register (USA) 2006a) and are being proposed elsewhere (United Nations Environmental Programme 2005, International Whaling Commission 2006, Panigada *et al*. 2006).

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Literature Cited

Aguilar, A. 2002. Fin whale *Balaenoptera physalus*. Pages 435–438 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

Best, P. B., V. M. Peddemors, V. G. Cockcroft and N. Rice. 2001. Mortalities of right whales and related anthropogenic factors in South Africa waters, 1963–1998. Cetacean Research and Management (Special Issue) 2:171–176.

Caswell, H., M. Fujiwara and S. Brault. 1999. Declining survival probability threatens the North Atlantic right whale. Proceedings of the National Academy of Sciences of the United States of America 96:3308–3313.

Clapham, P. J. 2002. Humpback whale *Megaptera novaeangliae*. Pages 589–592 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

Federal Register (USA). 2004. Advance notice of proposed rulemaking (ANPR) for right whale ship strike reduction. 1 June 2004. 69:30857–30864.

Federal Register (USA). 2006. Proposed rule to implement speed restrictions to reduce the threat of ship collisions with North Atlantic right whales. 26 June 2006. 71:36299–36313.

Ford, J. K. B. 2002. Killer whale *Orcinus orca*. Pages 669–676 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

Fujiwara, M., and H. Caswell. 2001. Demography of the endangered North Atlantic right whale. Nature 414:537–541.

Gallos, L. K., and P. Argyrakis. 2001. Accurate estimation of the survival probability for trapping in two dimensions. Physical Review E 64:1–6.

Hain, J. 1997. Detectability of right whales in Cape Cod Bay relative to NE EWS monitoring flights. Pages 113–118 *in* A. Knowlton, S. Kraus, D. Meck and M. Mooney-Sues, eds. Shipping/Right Whale Workshop Report 97–103, The New England Aquarium, Boston.

Horwood, J. 2002. Sei whale *Balaenoptera borealis*. Pages 1069–1071 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

International Maritime Organisation. 2003. New and amended traffic separation schemes. REF. T2/2.07, COLREG.2/Circ.52.

International Whaling Commission. 2006. Ship strikes working group first progress report to the Conservation Committee. 58th Annual Meeting of the International Whaling Commission. Agentina item 4.2. IWC/58/CC3. May 2006. 22 pp.

Jensen, A. S., and G. K. Silber. 2003. Large whale ship strike database. U.S. Department of Commerce. National Oceanic and Atmospheric Administration. Technical Memorandum NMFS-OPR-. 37 pp.

Jones, M. J., and S. L. Swartz. 2002. Gray whale *Eschrichtius robustus*. Pages 524–536 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

Kato, H. 2002. Bryde's whales *Balaenoptera edeni* and *B. brydei*. Pages 171–177 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

Kenney, R. D. 2002. North Atlantic, North Pacific, and southern right whales *Eubalaena glacialis, E. japonica,* and *E. australis*. Pages 806–813 *in* W. F. Perrin, B. Würsig and J.

G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

KNOWLTON, A. R., AND S. D. KRAUS. 2001. Mortality and serious injury of northern right whales (*Eubalaena glacialis*) in the western North Atlantic Ocean. Journal of Cetacean Research and Management (Special Issue) 2:193–208.

KRAUS, S. D. 1990. Rates and potential causes of mortality in North Atlantic right whales (*Eubalaena glacialis*). Marine Mammal Science 6:278–291.

KRAUS, S. D., M. W. BROWN, H. CASWELL, C. W. CLARK, M. FUJIWARA, P. K. HAMILTON, R. D. KENNEY, A. R. KNOWLTON, S. LANDRY, C. A. MAYO, W. A. MCLELLAN, M. J. MOORE, D. P. NOWACEK, D. A. PABST, A. J. READ AND R. M. ROLLAND. 2005. North Atlantic right whales in crisis. Science 309:561–562.

KRAUS, S. D., P. K. HAMILTON, R. D. KENNEY, A. R. KNOWLTON AND C. K. SLAY. 2001. Reproductive parameters of the North Atlantic right whale. Journal of Cetacean Research and Management (Special Issue) 2:231–236.

LAIST, D. W., AND C. SHAW. 2006. Preliminary evidence that boat speed restrictions reduce deaths of Florida manatees. Mammal Science 22:472–479.

LAIST, D. W., A. R. KNOWLTON, J. G. MEAD, A. S. COLLET AND M. PODESTA. 2001. Collisions between ships and whales. Marine Mammal Science 17:35–75.

LOCKYER, C. 1976. Body weights of some species of large whales. Journal du Conseil International pour l'Exploration de la Mer 36:259–273.

MAYO, C. A., AND M.K. MARX. 1990. Surface foraging behaviour of the North American right whale, *Eubalaena glacialis*, and associated zooplankton characteristics. Canadian Journal of Zoology 68:2214–2220.

MOORE, M. J., A. R. KNOWLTON, S. D. KRAUS, W. A. MCLELLAN AND R. K. BONDE. 2004. Morphometry, gross morphology and available histopathology in North Atlantic right whale (*Eubalaena glacialis*) mortalities (1970–2002). Journal of Cetacean Research and Management 6:199–214.

MYERS, R. H., D. C. MONTGOMERY AND G. G. VINING. 2002. Generalized linear models with applications in engineering and the sciences. John Wiley & Sons, Inc, New York.

NOWACEK, D. P., M. P. JOHNSON AND P. L. TYACK. 2004. North Atlantic right whales (*Eubalaena glacialis*) ignore ships but respond to alerting stimuli. Proceedings of the Royal Society B 271:227–231.

PANIGADA, S., G. PESANTE, M. ZANARDELLI, F. CAPOULADE, A. GANNIER AND M. T. WEINRICH. 2006. Mediterranean whales at risk from fatal ship strikes. Marine Pollution Bulletin. 52:1287–1298.

PERRIN, W. F., AND R. L. BROWNELL. 2002. Minke whales *Balaenoptera acutorostrata* and *B. bonaerensis*. Pages 750–754 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

R FOUNDATION FOR STATISTICAL COMPUTING. 2005 R: A language and environment for statistical computing. R Development Core Team, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org.

SEARS, R. 2002. Blue whale *Balaenoptera musculus*. Pages 112–116 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

TERHUNE, J. M., AND W. C. VERBOOM. 1999. Right whales and ship noise. Marine Mammal Science 15:256–258.

TRITES, A. W., AND D. PAULY. 1998. Estimating mean body masses of marine mammals from maximum body lengths. Canadian Journal of Zoology 76:886–896.

UNITED NATIONS ENVIRONMENTAL PROGRAMME. 2005. Convention on migratory species. 04 November 2005. UNEP/CMS/Res. 8.22, 4 pp.

WARD-GEIGER, L. I., G. K. SILBER, R. D. BAUMSTARK AND T. L. PULFER. 2005. Characterization of ship traffic in right whale critical habitat. Coastal Management 33:263–278.

WHITEHEAD, H. 2002. Sperm whale *Pyseter macrocephalus*. Pages 1165–1172 *in* W. F. Perrin, B. Würsig and J. G. M. Thewissen, eds. Encyclopedia of marine mammals. Academic Press, San Diego, CA.

WILEY, D. N., R. A. ASMUTIS, T. D. PITCHFORD AND D. P. GANNON. 1995. Stranding and mortality of humpback whales, *Megaptera novaeangliae*, in the mid-Atlantic and southeast United States, 1985-1992. Fishery Bulletin U.S. 93:196–205.

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APPENDIX: ONE-DIMENSIONAL COLLISIONS WITHIN THE LIMITS OF THE ELASTIC AND INELASTIC EXTREMES (SEE ONLINE SUPPLEMENTARY MATERIAL FOR GREATER DETAIL)

Nomenclature

In all equations below, subscript 1 refers the vessel and subscript 2 refers to the whale. The prime indicates the respective postcollision momenta and velocities. The delta (Δ) indicates the change in either momentum ($\Delta \boldsymbol{p}$) or time (Δt), and boldface indicates vector quantities.

Newton's Second Law is

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}, \tag{1}$$

where $\boldsymbol{F}$ is force, $\boldsymbol{p} = m\boldsymbol{v}$ is the momentum; the product of mass (m) and velocity ($\boldsymbol{v}$).

Conservation of Linear Momentum: When no net external force acts on a system, the total linear momentum of the system cannot change, thus,

$$m_1\boldsymbol{v}_1' + m_2\boldsymbol{v}_2' = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2. \tag{2}$$

One-dimensional elastic collision:

An elastic collision is one where the postcollision kinetic energy of the system is equal to the precollision kinetic energy of the system

$$\frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2, \tag{3}$$

which with Eq. 2, yields

$$v'_2 - v'_1 = -(v_2 - v_1). \tag{4}$$

Hence, for elastic collisions the relative speed of recession postcollision equals the relative speed of approach precollision.

Using Eq. 2 and Eq. 4, the postcollision velocity of the whale is solved as:

$$v'_2 = \frac{2m_1v_1 + m_2v_2 - m_1v_2}{m_1 + m_2}. \tag{5}$$

Substituting Eq. 5 into the momentum term of Eq. 1 yields

$$F = \frac{dp}{dt} \doteq \frac{\Delta p}{\Delta t} \doteq \frac{p'_2 - p_2}{\Delta t} \doteq \frac{m_2\left(\dfrac{2m_1 v_1 + m_2 v_2 - m_1 v_2}{m_1 + m_2}\right) - m_2 v_2}{\Delta t}. \quad (6)$$

One-dimensional inelastic collision: A perfectly inelastic collision is one where only the momentum of the system is conserved and the postcollision velocities of the two colliding bodies are equal and move as one body at velocity v'(*i.e.*, $v' = v'_1 = v'_2$). By using Eq. 2, the postcollision velocity is defined as:

$$v' = \frac{m_1 v_1 + m_2 v_2}{(m_1 + m_2)}. \quad (7)$$

Substituting Eq. 7 into the momentum term in Eq. 1 yields

$$F = \frac{dp}{dt} \doteq \frac{\Delta p}{\Delta t} \doteq \frac{p'_2 - p_2}{\Delta t} \doteq \frac{m_2\left(\dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right) - m_2 v_2}{\Delta t}. \quad (8)$$

Assumptions for the one-dimensional limiting cases first approximations: For both types of collisions above, elastic and perfectly inelastic, we can reasonably assume that both the mass and velocity of a large whale are much less than for a vessel; that is, $m_1 \gg m_2$ and $v_1 \gg v_2$. With these assumptions, the force equations (Eq. 6 and Eq. 8) above simplify to
the elastic extreme

$$F \approx \frac{2m_2}{\Delta t} v_1 \text{ if } \frac{v_2}{v_1} \ll 1 \quad \text{and} \quad \frac{m_2}{m_1} \ll 1, \quad (9)$$

and the perfectly inelastic extreme

$$F \approx \frac{m_2}{\Delta t} v_1 \text{ if } \frac{v_2}{v_1} \ll 1 \quad \text{and} \quad \frac{m_2}{m_1} \ll 1. \quad (10)$$

Thus, in either case, the forces involved in the collision are the product of the mass of the whale and the speed of the vessel.

Supplementary Material

The following supplementary material is available for this article online:
One-dimensional collisions within the limits of the elastic and inelastic extremes.